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Estimation of elastic constants from ellipsoidal velocities in orthorhombic media.

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Summary

This paper introduces an ellipsoidal approximation of phase and group velocities of the P-, $S1$- and $S2$-wave propagation modes in an orthorhombic media and, show how to estimate elastic constants from these velocities. The procedure is basically two-fold. First, we estimate seven ellipsoidal velocities near the vertical symmetry axis which represent both the direct and the NMO velocities (in the symmetry planes XZ and YZ) for each wave propagation mode. Secondly, the ellipsoidal velocities are used to build a linear square system whose simple analytical solution returns an estimation of the elastic constants. The accuracy of this inversion process depends on how accurately NMO velocities are estimated. The whole procedure is valid for homogeneous media, although extensions to heterogeneous media can be performed through tomographic techniques.

Introduction

An orthorhombic model describes a layered medium fractured in two orthogonal directions. Wave propagation in orthorhombic media has been extensively studied; see e.g., Cheadle et al. (1991); Grechka and Tsvankin (1996); Schoenberg and Helbig (1991); Tsvankin (1996a); Tsvankin (1996b) among others. Recently, Tsvankin (1996b) analyzed the behavior of P-waves within a weak anisotropic media for an orthorhombic model. In his work, Tsvankin stresses out the importance of having different approximations to the wave velocities in order to either invert elastic parameters or perform velocity analysis in complex structures. However, his approach does not deal with the shear wave propagation.

We show in this paper that the square of phase velocities for orthorrombic media near the vertical symmetry axis can be approximated by an ellipsoidal function. We establish that this ellipsoidal function in the phase domain corresponds to ellipsoidal functions in the group domain. This generalizes to orthorrombic media the results obtained by Levin (1978); Byun (1982); Muir (1990) for TI media.

The ellipsoidal approximation is a useful device that allows to go back and forth between phase and group physical entities. In this way, we are able not only to characterize media along a vertical symmetry axis but also to estimate the elastic constants describing the anisotropic medium.

Michelena (1994) showed that the elastic constants controlling compressional and shear wave propagation can be estimated from P- and SV-wave traveltimes from either crosswell or VSP geometries. The present approach extends his work for orthorhombic media.

Both the ellipsoidal approximation and the inverse mapping to estimate elastic constants are valid for arbitrary strengths of anisotropy, but are restricted to a near symmetry axis, unlike the weak anisotropy approximation of Tsvankin (1996a) for orthorhombic media, which is suitable only for a wider angle of P-wave propagation.

Ellipsoidal velocities near the vertical axis

The eigenvalues of the Christoffel equation are given by the roots of the characteristic polynomial

$$\det\left[G_{ik}\left(\theta_1,\theta_2\right) - W\delta_{ik}\right] = 0, \qquad (1)$$

of cubic-order in W. Here, $W = \rho V^2$, is the square of the phase velocity (assuming $\rho \equiv 1$); δ_{ik}, is the Kronecker Delta-function; $G_{ik}\left(\theta_1,\theta_2\right)$, is the Christoffel matrix determined by the elastic constants depending on the polar angle, θ_1, and on the azimuthal angle, θ_2 (see Figure 1). The roots of the cubic polynomial represent the square phase velocities for each of the different wave propagation modes, namely $P, S1$ and $S2$. The exact group velocity can be found numerically by

$$V_{g_i} = \frac{C_{ijkl}\alpha_i\alpha_k\beta_l}{W_i^{1/2}}, \qquad (2)$$

where W_i is the squared phase velocity for each of the different modes $i = P, S1, S2$; C_{ijkl} denotes the constant elastic tensor; the α_1 and α_2, are the eigenvectors of the Christoffel equation and β_l is the normal direction.

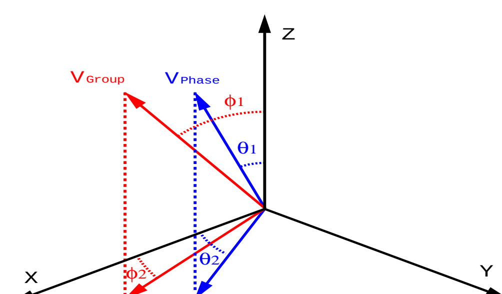

Fig. 1: Polar and azimuthal angles.

In order to understand the behavior of the phase velocity for a small polar angle, θ_1, the square of the phase velocity, $W_i\left(\theta_1,\theta_2\right)$ is expanded by Taylor series for a small θ_1 and a fixed θ_2. Neglecting terms higher than $\sin^2\theta_1$, we obtain

$$\begin{aligned} W_i\left(\theta_1,\theta_2\right) &= W_i\left(0,\theta_2\right) \\ &+ \frac{\partial W_i}{\partial \sin^2\theta_1}\left(0,\theta_2\right)\sin^2\theta_1. \end{aligned} \qquad (3)$$

Note that no approximation is performed in the azimuthal angle, θ_2. The first term on the right hand side of (3) and the derivative term are computed from the characteristic polynomial (1) at θ_1 and at a fixed θ_2. Hence, terms for the squared phase velocities W_P, W_{S1} and W_{S2} are given by Contreras et al. (1997):

$$\begin{aligned} W_i\left(\phi_{1,i}, \theta_{2,i}\right) &= W_{Z,i} \cos^2 \theta_{1,i} \\ &+ W_{\mathrm{NMO}[XZ],i} \cos^2 \theta_{2,i} \sin^2 \theta_{1,i} \\ &+ W_{\mathrm{NMO}[YZ],i} \sin^2 \theta_{2,i} \sin^2 \theta_{1,i}, \end{aligned} \tag{4}$$

for $i = P, S1$ and $S2$. Equation (4) represents an ellipsoid in the phase domain. The propagation along the vertical for each wave-mode is given by $W_{Z,P} = C_{33}$, $W_{Z,S1} = C_{44}$ and $W_{Z,S2} = C_{55}$. The NMO velocities along the symmetry planes are expressed as follows:

$$W_{\mathrm{NMO}[XZ],P} = C_{55} + \frac{(C_{13}+C_{55})^2}{C_{33}-C_{55}}, \tag{5}$$

$$W_{\mathrm{NMO}[YZ],P} = C_{44} + \frac{(C_{23}+C_{44})^2}{C_{33}-C_{44}}, \tag{6}$$

$$W_{\mathrm{NMO}[YZ],S1} = C_{22} + \frac{(C_{23}+C_{44})^2}{C_{44}-C_{33}}, \tag{7}$$

$$W_{\mathrm{NMO}[XZ],S2} = C_{11} + \frac{(C_{13}+C_{55})^2}{C_{55}-C_{33}}, \tag{8}$$

and

$$W_{\mathrm{NMO}[XZ],S1} = W_{\mathrm{NMO}[YZ],S2} = C_{66}. \tag{9}$$

The above expressions can be employed to estimate the elastic constants as a function of phase velocities in an orthorhombic media. However, in general, phase velocities are hard to obtain from traveltimes described by group velocities. In order to generate expressions for group velocities with the ellipsoidal approximation at small polar angles, we rely on a transformation similar to the one proposed by Levin (1978); Byun (1982); Muir (1990). In the 3-D orthorhombic case, their transformation can be simply recast as

$$\begin{aligned} \theta_{1,i} &\rightarrow \phi_{1,i} \\ \theta_{2,i} &\rightarrow \phi_{2,i} \\ W_i &\rightarrow W_{g_i}^{-1} \end{aligned} \tag{10}$$

where $\phi_{1,i}$, is the exact polar group angle and $\phi_{1,i}$, is the azimuthal group angle both computed from equation (2) for $i = P, S1, S2$ (see Figure 1). The mapping just shown suggests a direct correspondence between each phase and group entity, in the same fashion that trivially occurs when the model is isotropic, elliptically anisotropic (in TI media) or when the ray travels along a particular axis of symmetry.

Applying transformation (10) to equations (5-9) the corresponding ellipsoidal group velocities for each wave mode propagation near the vertical axis are

$$\begin{aligned} W_{g_i}^{-1}\left(\phi_{1,i}, \phi_{2,i}\right) &= W_{Z,i}^{-1} \cos^2 \phi_{1,i} \\ &+ W_{\mathrm{NMO}[XZ],i}^{-1} \cos^2 \phi_{2,i} \sin^2 \phi_{1,i} \\ &+ W_{\mathrm{NMO}[YZ],i}^{-1} \sin^2 \phi_{2,i} \sin^2 \phi_{1,i}, \end{aligned} \tag{11}$$

for $i = P, S1$ and $S2$, which represents an ellipsoid in the group domain.

It is worth to add that the above approximation is valid not only near the vertical symmetry axis Z. It also applies to velocities near the horizontal axis X or Y. For the X axis, we need to make the following change of subindexes: $C_{11} \leftrightarrow C_{33}$, $C_{12} \leftrightarrow C_{23}$, $C_{44} \leftrightarrow C_{66}$. For the Y axis follows: $C_{12} \leftrightarrow C_{13}$, $C_{22} \leftrightarrow C_{33}$, and $C_{55} \leftrightarrow C_{66}$. The rest of the elastic constants remain unchanged.

Inverse mapping from ellipsoidal velocities to elastic constants

The formulae derived in the previous section can be employed to find out the elastic constants controlling each wave propagation mode near an axis of symmetry (in analogy to the VTI case analized by Michelena (1994)). To do so, it is important to be aware of the geometry of the problem, the aperture of both polar and azimuthal angles and the propagation mode.

By using P-,S1- and S2- full aperture traveltime measurements, it is possible to estimate the nine elastic constants describing an orthorhombic medium from lab measurements Cheadle et al. (1991). This estimation results after solving a nonlinear system of equations (the Christoffel stiffness equations) that simplifies to a diagonal system when wave propagation occurs along a symmetry axis. That is: $C_{11} = W_{X,P}$, $C_{22} = W_{Y,P}$, $C_{33} = W_{Z,P}$, $C_{44} = W_{Z,S1} = W_{Y,S1}$, $C_{55} = W_{X,S1} = W_{Z,S2}$ and $C_{66} = W_{Y,S2} = W_{X,S2}$. However, this method is hardly applicable in practical situations since it requires wide aperture data. Clearly ,this requirement does not exist for surface, crosswell or VSP geometries.

In our case, seven elastic constants can be determined from the set of NMO velocity equations implied by (5-8) and the direct velocities $W_{Z,P}$, $W_{Z,S1}$ and $W_{Z,S2}$. Evaluating the equations for a fixed angle θ_1 and a pair of different values for θ_2 we obtain 7 equations with 7 unknowns. The independent terms are given by: $W_{Z,P}$, $W_{Z,S1}$, $W_{Z,S2}$, $W_{\mathrm{NMO}[XZ],P}$, $W_{\mathrm{NMO}[YZ],P}$, $W_{\mathrm{NMO}[YZ],S1}$ and $W_{\mathrm{NMO}[XZ],S2}$.

The explicit analytic solution of this system is given by

$$\begin{aligned} C_{33} &= W_{Z,P}, \quad C_{44} = W_{Z,S1}, \quad C_{55} = W_{Z,S2}, \\ C_{11} &= W_{NMO[XZ],S2} + W_{NMO[XZ],P} - W_{Z,S2}, \\ C_{22} &= W_{NMO[YZ],S1} + W_{NMO[YZ],P} - W_{Z,S2}, \\ C_{13} &= \sqrt{(W_{NMO[XZ],P} - W_{Z,S2})(W_{Z,P} - W_{Z,S2})} \\ &- W_{Z,S2}, \\ C_{23} &= \sqrt{(W_{NMO[YZ],P} - W_{Z,S1})(W_{Z,P} - W_{Z,S1})} \\ &- W_{Z,S1}. \end{aligned}$$

The constant C_{12} is not computed above since it controls the propagation along the horizontal XY plane which is not well approximated by the ellipsoidal mapping. Note that the constant C_{66} can be immediately attained from (9), where the NMO and direct velocities are the same. In contrast to Cheadle et al. (1991), both C_{11} and C_{22} are

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linear combinations of the ellipsoidal parameters lying on the symmetry planes and are independent of $W_{X,P}$ and $W_{Y,P}$.

Therefore, estimation of elastic constants in homogeneous orthorhombic media can be performed by fitting traveltimes near a single axis of symmetry with ellipsoidal models. In the case of a heterogeneous media, the inversion can be carried out by tomographic techniques. Such approach was previously proposed by Michelena et al. (1995) in TI media.

Numerical example

To validate our approximation, we performed the corresponding inversion on the Cracked Greenhorn Shale case used by Dellinger (1991), whose elastic constants are

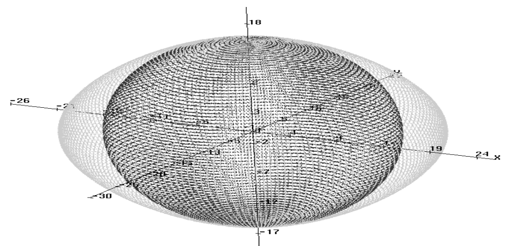

Fig. 2: Exact (gray) and ellipsoidal (black) impulse-response for P-wave.

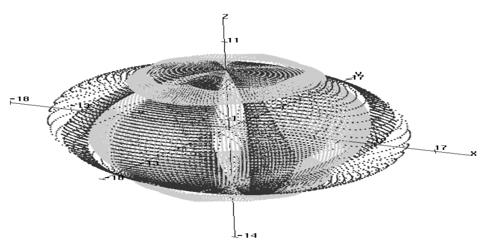

Fig. 3: Exact (gray) and ellipsoidal (black) impulse-response for $S1$-wave.

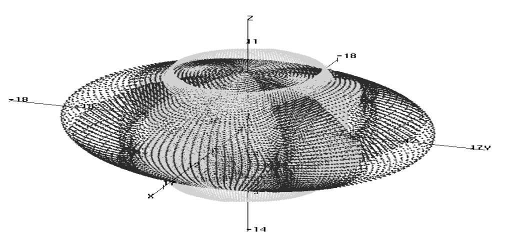

Fig. 4: Exact (gray) and ellipsoidal (black) impulse-response for $S2$-wave.

$$C_{ij} = \begin{pmatrix} 336.6 & 117.3 & 103.3 & 0.0 & 0.0 & 0.0 \\ 117.3 & 310.0 & 92.3 & 0.0 & 0.0 & 0.0 \\ 103.3 & 92.3 & 223.9 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 49.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 54.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 96.4 \end{pmatrix}.$$

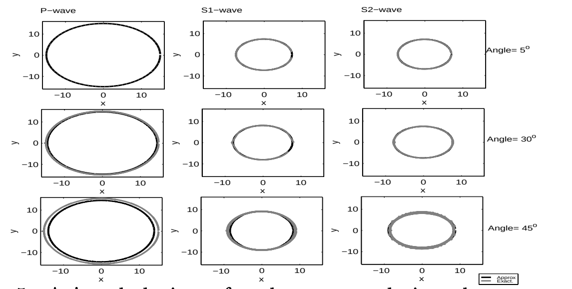

Fig. 5: Azimuthal view of each wave mode impulse-response for different polar group angles.

We can observe from Figures 2-4, how both the exact response and the approximated response coincide very well near the vertical axis. As the polar angle increases, their separation also increases monotonically until it reaches a maximum at 90^o. This means that the horizontal velocity is not well reproduced by the ellipsoidal approximation as expected. Figures 3-4 show anomalies (i.e., triplications) for intermediate polar angles for the exact $S1$- and $S2$- wave impulse-responses. These anomalies are unseen by the ellipsoidal approximation, which is a single-valued function.

Figure 5, shows horizontal slices of the 3-D impulse-response for each wave propagation mode at different polar angles. We can observe that exact and ellipsoidal group velocities are almost the same at the XY symmetry plane for a small vertical aperture. At a polar angle of 45^o we can observe how the ellipsoidal approximation begins to deteriorate. The maximum approximation error is reached at a polar angle of 90^o (not shown here).

Figures 6-9 show the relative error made in the estimation of the elastic constants from the NMO velocities. For small polar angles ($< 10^o$) the error is negligible regardless of azimuth. In fact, a pair of azimuthal angles (with one of them always at 0^o degrees) were computed in all cases. For polar angles between $10^o - 30^o$ the error is smaller in C_{11} and C_{22} than in C_{13} and C_{23}. The azimuth angle has no major influence in the accuracy of the inversion. At large angles ($> 30^o$) the error in C_{11} and C_{22} is greater than C_{13} and C_{23} and changes with respect azimuthal variation start to be noticeable for C_{22} and C_{23}.

Conclusions

We have established that the impulse-response of all different modes of wave propagation are ellipsoids for small polar angles (near the vertical axis). As a consequence, horizontal NMO velocities are ellipses in agreement with the results published by Grechka and Tsvankin (1996).

On the other hand, we have shown how to estimate elastic constants of homogeneous orthorhombic media from P-, $S1$- and $S2$- traveltimes near a single axis of symmetry. The accuracy of this estimation from ellipsoidal parameters depends on the accuracy of the estimation of the NMO velocities for small polar angles. The quality of the inversion does not depend on the azimuthal angle for small polar angles around the symmetry axis.

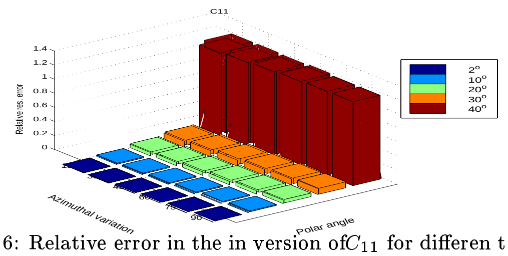

Fig. 6: Relative error in the inversion of C_{11} for different polar and azimuthal angles.

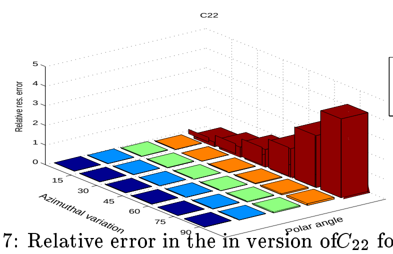

Fig. 7: Relative error in the inversion of C_{22} for different polar and azimuthal angles.

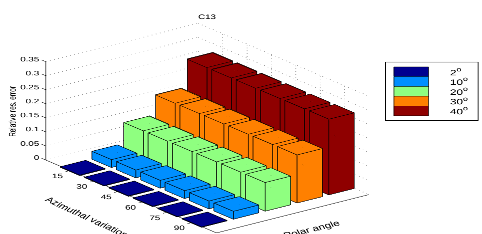

Fig. 8: Relative error in the inversion of C_{13} for different polar and azimuthal angles.

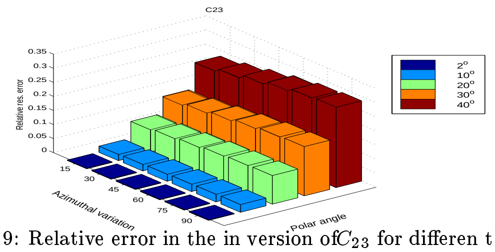

Fig. 9: Relative error in the inversion of C_{23} for different polar and azimuthal angles.

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