

SUMMARY

I propose a method of seismic traveltime tomography for azimuthally anisotropic media consisting of dipping transversely isotropic layers. The velocity model is elliptical with variable inclination of the axis of symmetry. When used with the double elliptic approximation, the method can be used to fit any wave type in transversely isotropic media. This method is a generalization of conventional tomographic inversion techniques where the interfaces of the model are allowed to change their positions and to be eliminated from the inversion as iterations proceed, which allows a better estimation of the remaining unknowns. Ray tracing is performed in anisotropic models. The technique is tested with synthetic and field data from a cross-well experiment where both strong anisotropy and strong velocity contrasts are present.

INTRODUCTION

The model space needed to perform seismic traveltime tomography consists of two separate models: one for the velocities and one for the heterogeneities. The selection of each of these models should be made depending on any prior knowledge we may have about the true medium. Even if we don't know anything about the medium, the selection should be guided by the type of information we want to extract from the data.

From the inversion point of view, velocity and heterogeneities are closely related. When the model for the velocities is isotropic but the real medium is not, the image of heterogeneities obtained is distorted. On the contrary, if we want to estimate velocity anisotropy in the medium but the heterogeneities are not properly described, we might get wrong estimates for the velocities. These problems can be solved by describing properly both velocities and heterogeneities and then estimating both *simultaneously*. However, this might be difficult to do for a general medium.

Velocity anisotropy, heterogeneity, or both at the same time, can affect the traveltimes nonlinearly depending on how strong they are and how complex the model that describes them is. When one of these nonlinearities is neglected, the computations related to the other one can be simplified. For example, assuming weak anisotropy, Pratt and Chapman (1991) and Chapman and Pratt (1991) proposed to do the ray tracing in isotropic media and to use those rays to invert in the anisotropic model. Michelena et al. (1992) simplify the problem even more and present examples where the velocity contrasts are small enough such that the rays traced on the elliptically anisotropic models are straight and the only nonlinearity that remains is in the estimation of the anisotropy.

The simplest anisotropic velocity model we can assume is elliptical. Even though it is not a good approximation for velocities at all angles in a transversely isotropic media, elliptical anisotropy is a good *paraxial* approximation that can be used in two steps (once around the horizontal and once around the vertical) to approximate globally the slowness surface and impulse response of any wave type in a transversely isotropic medium (Muir (1990) and Dellinger and Muir (1991)). Because the approximation uses two perpendicular ellipses, it is called double elliptic approximation. What is needed in order to use this approximation is an inversion procedure that fits traveltimes with ellipses, once a model for heterogeneities has been chosen.

The simplest model to describe the heterogeneities of the medium is 1D. If dips are present, we can allow the interfaces of the 1D model to have variable slope. However, if the spatial variations in the medium are more complex, we may need to use models that specifically account for them, for example a fine 2D grid.

The model for velocities and heterogeneities I will use in this paper compromises between the simplicity of 1D isotropic and complexity of 2D anisotropic models (Michelena et al., 1992). This compromise addresses three issues: how general the medium can be, how stable the inversion procedure is and how difficult the ray tracing is. Such an "intermediate" model consists of homogeneous elliptically anisotropic blocks separated by straight interfaces of variable dip and intersect. Interfaces may change position during the inversion process but are never allowed to cross in the area of interest. In Michelena (1992), I show how to trace rays in this type of model.

Certain types of azimuthally anisotropic media can be also approximated with this model for velocities and heterogeneities, in particular those formed by dipping transversely isotropic layers. This is done by considering also the inclination of the axis of symmetry as a variable.

A theoretical discussion of the technique is followed by examples with cross-well synthetic data and field data from the BP's Devine test site.

FORWARD MODELING

The traveltime for a ray that travels a distance d in an homogeneous medium with elliptical anisotropy and axis of symmetry forming an angle γ with respect to the vertical (Figure 1) is

$$t = \left(\frac{(\Delta x \cos \gamma + \Delta z \sin \gamma)^2}{V_{\perp}^2} + \frac{(-\Delta z \cos \gamma + \Delta x \sin \gamma)^2}{V_{\parallel}^2} \right)^{1/2}, \tag{1}$$

where $\sqrt{\Delta x^2 + \Delta z^2} = d$ and V_{\parallel} and V_{\perp} are the velocities in the directions parallel and perpendicular respectively to the axis of symmetry. In Michelena (1992), I explain how to derive this expression.

Figure 2 shows the type of model that I will consider in this paper. It consists of homogeneous elliptically anisotropic blocks separated by straight interfaces of variable dip (a_j) and intercept (b_j). The expression for the traveltimes $t_{i,j}$ of the i^{th} ray in the j^{th} cell is

$$t_{i,j} = \sqrt{\Delta X_{i,j}^2 S_{\perp,j}^2 + \Delta Z_{i,j}^2 S_{\parallel,j}^2}, \quad (2)$$

where $S_{\perp,j}$, $S_{\parallel,j}$, $\Delta X_{i,j}$ and $\Delta Z_{i,j}$ are

$$S_{\perp,j} = \frac{1}{V_{\perp,j}},$$

$$S_{\parallel,j} = \frac{1}{V_{\parallel,j}},$$

$$\Delta X_{i,j} = \Delta x_{i,j} \cos \gamma_j + (a_{j+1} x_{i,j+1} + b_{j+1} - a_j x_{i,j} - b_j) \sin \gamma_j,$$

$$\Delta Z_{i,j} = -(a_{j+1} x_{i,j+1} + b_{j+1} - a_j x_{i,j} - b_j) \cos \gamma_j + \Delta x_{i,j} \sin \gamma_j.$$

In the previous expressions, $(x_{i,j}, z_{i,j})$ is the point of intersection between the i^{th} ray and the j^{th} interface. $\Delta x_{i,j}$ is defined as $\Delta x_{i,j} = x_{i,j+1} - x_{i,j}$.

The total traveltimes for a ray that travels from source to receiver is

$$t_i(\mathbf{m}) = \sum_{j=1}^N t_{i,j}(\mathbf{m}) \quad i = 1, \dots, M, \quad (3)$$

where \mathbf{m} is the vector of model parameters of $5N$ elements:

$$\mathbf{m} = (m_1, \dots, m_N, m_{N+1}, \dots, m_{2N}, m_{2N+1}, \dots, m_{3N}, m_{3N+1}, \dots, m_{4N}, m_{4N+1}, \dots, m_{5N})$$

$$= (S_{\perp,1}, \dots, S_{\perp,N}, S_{\parallel,1}, \dots, S_{\parallel,N}, \gamma_1, \dots, \gamma_N, b_1, \dots, b_N, a_1, \dots, a_N),$$

and M is the total number of traveltimes.

Equation (3) is the system of nonlinear equations that relates the model parameters with the measured traveltimes. A linearized version of this equations will be used in the next section to solve the inverse problem.

INVERSE MODELING

When solving the inverse problem, the goal is to estimate two different sets of coupled unknowns: the model parameters and the ray paths. The usual way to decouple them is by invoking Fermat's principle, which "justify" the trick of assuming one to estimate the other in an iterative fashion, as long as the magnitude of the changes from one step to the next are kept small.

Once the ray paths have been estimated, the system of nonlinear equations (3) needs to be solved in order to find a new model where rays are going to be traced again. One way to do this is as a sequence of linearized steps, starting from a given initial model \mathbf{m}_0 . The first step is to approximate (3)

by its first order Taylor series expansion centered in a given model \mathbf{m}_0 :

$$t_i(\mathbf{m}) \approx t_i(\mathbf{m}_0) + \nabla t_i(\mathbf{m}_0) \cdot (\mathbf{m} - \mathbf{m}_0)$$

$$= t_i(\mathbf{m}_0) + \sum_{n=1}^{5N} \tilde{\mathbf{J}}_{in} (m_n - m_{0n}), \quad (4)$$

where the elements of the Jacobian $\tilde{\mathbf{J}}_{in}$ are

$$\tilde{\mathbf{J}}_{in} = \left. \frac{\partial t_i}{\partial m_n} \right|_{\mathbf{m}=\mathbf{m}_0}.$$

The explicit form of these derivatives can be easily calculated from equation (3).

If we assume that $t_i(\mathbf{m})$ represents one component of the vector \mathbf{t} of measured traveltimes, we can compute the perturbations $\Delta \mathbf{m}_n = (m_n - m_{0n})$ once the traveltimes in the reference model \mathbf{m}_0 has been calculated. The perturbation $\Delta \mathbf{m} = (\mathbf{m} - \mathbf{m}_0)$ is the solution of the following system of equations

$$\tilde{\mathbf{J}} \Delta \mathbf{m} = \Delta \mathbf{t} \quad (5)$$

where $\Delta t_i = t_i(\mathbf{m}) - t_i(\mathbf{m}_0)$. This system of linear equations will be solved using an LSQR variant of the conjugate gradients algorithm.

In practice, only a fraction r of the correction $\Delta \mathbf{m}$ is added to the given model

$$\mathbf{m}_1 = (\mathbf{m}_0 + r \Delta \mathbf{m})$$

where r (the step length) is usually small to avoid large changes in the ray paths from one iteration to the next.

Constraints

Since the interfaces are allowed to change position from one iteration to the next, the algorithm has to make sure that each new model does not contain crossing interfaces in the area of interest. If two interfaces cross after adding $r \Delta \mathbf{m}$ to the given model, the step length r is reduced until those interfaces do not cross after the correction. Once the model has been updated and there are no crossing interfaces, the algorithm checks whether thin layers have been created or not and if so, those layers are eliminated from the inversion, reducing by a multiple of 5 the number of model parameters (five parameters per each eliminated layer).

SYNTHETIC EXAMPLE

4992 traveltimes were generated through the model shown in Figure 3 for a geometry where sources were located within a range of ± 45 degrees with respect to each receiver position. The ray tracing algorithm described in Michelena (1992) was used to compute the synthetic traveltimes and to trace the rays needed in the nonlinear inversion.

The starting model used for inversion was homogeneous and isotropic, described by 17 horizontal layers of equal thickness. The inclination used for the axis of symmetry was

$\gamma = 0$ in all layers. Figure 4 shows the initial positions of the boundaries in the starting model. By starting the iterations with this model I wanted to test how the interfaces arrange themselves to create a dipping layer not present in the initial model. The inversion was constrained by not allowing parallel layers (within ± 5 degrees) to be thinner than 15 ft. When this condition was met, the corresponding layer was eliminated, reducing the number of model parameters. No smoothing was applied to the model after each iteration.

Figure 5 shows the result of the inversion. Notice how boundaries have changed positions with respect to their initial values. Notice also that the initial 17 layers were reduced to 10 to allow the positioning of the dipping ones at the correct depths with the correct dips. The inclination of the axes of symmetry estimated by the algorithm is also correct.

FIELD DATA EXAMPLE

The technique previously described was applied to a cross-well data set recorded at the BP's Devine test site, located southwest of San Antonio, in Texas. A sketch of the geology at this site is shown in Figure 6. This test site, first cited by Harris (1988), has been cited in recent publications to illustrate the application of different techniques. Two of these publications address the problem of estimation of velocity anisotropy from cross-well data (Miller and Chapman, 1991; Onishi and Harris, 1991).

P-waves with frequencies between 200 Hz and 4000 Hz were recorded between two cased boreholes (Wilson 2 and Wilson 4) whose separation at the surface was 330 ft. Receivers were separated 10 ft and sources 20 ft. Figure 6 also shows the corresponding sonic logs at each well.

1660 traveltimes were picked from a small data set of only 26 gathers. From these traveltimes only those corresponding to angles less than 45 degrees between source and receiver were kept for the inversion.

The starting model (not shown) consisted of 140 horizontal layers 5 ft thick. The velocity for all layers was 12000 ft/sec. Layers thinner than 1 ft were not allowed in the model. The inversion was not constrained to match the vertical velocities or depth of certain layers using information derived from the sonic logs, although the information about horizontal dips is already present in the initial model.

The inversion produced a model with horizontal layers (all dips less than 0.1 degree) and vertical axes of symmetry. Therefore, $V_1 = V_x$ and $V_3 = V_z$. Figure 7 shows the horizontal and vertical velocities as well as an average sonic log from the two wells, blocked every 7 ft. The first thing we notice from Figure 7 is that $V_x \geq V_z$ for almost all depths. We also notice that V_z is much closer to the sonic log than V_x , which is more than 30% larger than the log velocity in the shale and clay intervals. The vertical velocity contrast between limestone and shale is greater than 70%. Still, the inversion does a good job in estimating V_z . For this model, the average absolute value of the residuals is 0.2 ms. 12 layers were eliminated during the inversion procedure.

These results agree with the ones presented by Miller and Chapman (1991) and Onishi and Harris (1991).

Figure 7 also shows that the elliptical velocity model explains most of the *P*-wave anisotropy at this site. However, note that the estimated vertical velocity (NMO velocity) is in general smaller than the log velocity, which indicates that the elliptical model is not fully adequate to describe the possible transverse isotropic nature of this medium. 2D variations in the medium not described correctly by the model of heterogeneities might explain these and other differences.

CONCLUSIONS

A tomographic inversion technique that fits seismic traveltimes with elliptical velocity functions have been presented. The elliptical parameters estimated with this inversion can be plugged into the double elliptic approximation to estimate more general transversely isotropic models, when data from different geometries is used.

Since the inclination of the axis of symmetry is also a variable in the inversion procedure, certain types of azimuthally anisotropic media can be approximated, in particular those formed by dipping transversely isotropic layers.

The model for heterogeneities is described as a superposition of homogeneous orthogonal regions whose boundaries may change their positions as iterations proceed but they are not allowed to cross in the area of interest. When two parallel interfaces move too close to each other, one of them (along with its upper interval) is eliminated from the inversion, reducing in this way the number of unknowns.

The technique was successfully applied to synthetic and field data where both strong anisotropy and strong velocity contrasts were present.

ACKNOWLEDGMENTS

I would like to thank BP Exploration for providing the field data set used in this paper. These data were acquired by Jerry M. Harris while he was at BP. I thank him and also Francis Muir, who have been a source of important and challenging suggestions that have enormously contributed to my research. Spyros Lazaratos picked the first arrival traveltimes. Finally, I would like to thank INTEVEP, S.A for financial support.

REFERENCES

- Chapman, C. H. and Pratt, R. G., 1991, Traveltime tomography in anisotropic media: theory: submitted to *Geophys. J. Intl.*
- Dellinger, J. and Muir, F., 1991, The double elliptic approximation in the group and phase domains: *SEP-70*, 361-366.
- Harris, J. M., 1988, Cross-well seismic measurements in sedimentary rocks: presented at the 58th Ann. Internat. Mtg., Soc. Expl. Geophys.

Miller, D. E., and Chapman, C. H., 1991, Incontrovertible evidence of anisotropy in cross-well data: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Michelena, R.J., 1992, Traveltime tomography in azimuthally anisotropic media: STP-3.

Michelena, R.J., 1992, Kinematic ray tracing in anisotropic layered media: STP-3.

Michelena, R.J., Muir, F. and Harris, J. M., 1992, Anisotropic traveltime tomography: submitted to publication in Geophysical Prospecting.

Muir, F., 1990, A modified anisotropic system: SEP-67, 41-42.

Onishi, M., and Harris, J., M., 1991, Anisotropy from head waves in cross-well data: presented at the 61st Ann. Internat. Mtg., Soc. Expl. Geophys.

Pratt, R. G., Chapman, C. H., 1991, Traveltime tomography in anisotropic media: application: submitted to Geophys. J. Intl.

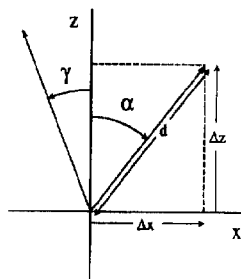


Figure 1: Ray traveling a distance d in a medium with tilted axis of symmetry. α and γ are the angles of the ray and the axis of symmetry respectively with respect to the vertical.

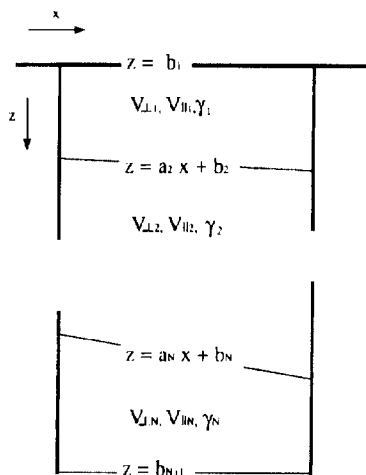


Figure 2: Model of velocities and heterogeneities. The top and bottom interfaces are horizontal ($a_1 = a_{N+1} = 0$) and located at known depths.

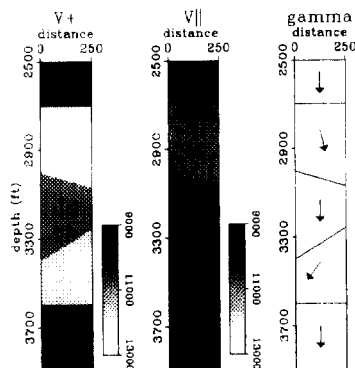


Figure 3: Synthetic model used to test the inversion. The third interface is dipping 15 degrees and the fourth one -30. The inclination of the axis of symmetry in the second layer is -15 degrees and 40 degrees in the fourth layer. The ratio V_{\perp}/V_{\parallel} at the fourth layer is 1.26. The gray scale shows variations in velocity. "V+" stands for V_{\perp} , "V||" for V_{\parallel} and "gamma" for γ .

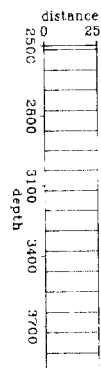


Figure 4: Initial position of the interfaces in the starting model (homogeneous isotropic). The velocity is equal to 12000 ft/sec and the inclination of the axes of symmetry with respect to the vertical is zero for all layers.

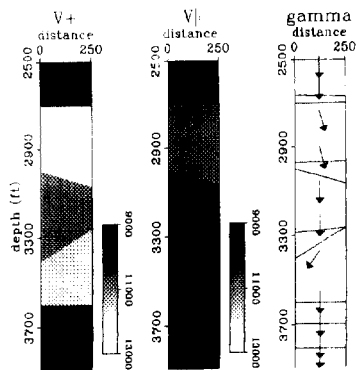


Figure 5: Result of the inversion. Notice how the interfaces have changed their initial positions. The position of each arrow's head shows the interval that correspond to each γ . The gray scale shows variations in velocity.

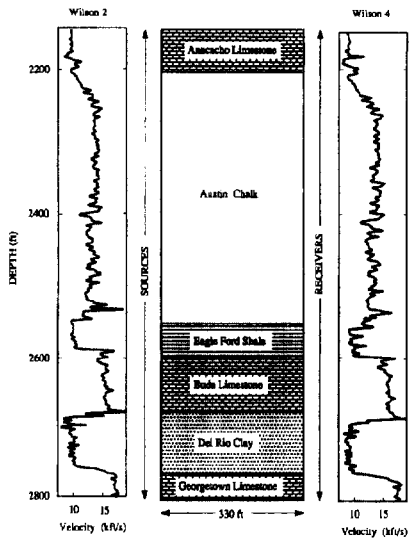


Figure 6: Sketch of the geology at the Devine test site with the sonic logs at each well. Although mostly flat layered, small 2D variations can be seen.

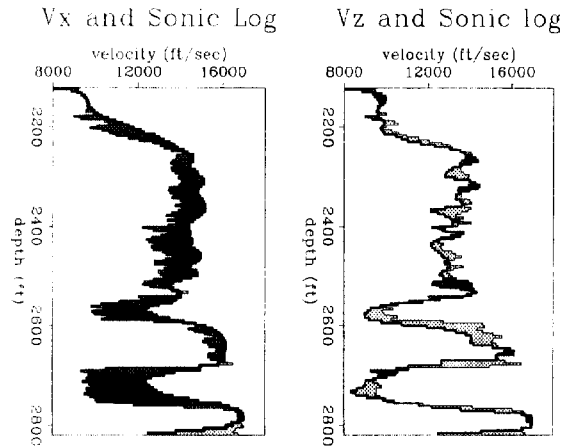


Figure 7: Result of the inversion. An average sonic log blocked every 7 ft is compared with the two estimated velocities. The vertical velocity is closer and better correlated to the log. Notice how the amount of anisotropy changes through the model, reaching a peak at the shale and clay intervals. The model is described by 128 layers.