Tomographic Estimation of Elastic Constants in Heterogeneous Transversely Isotropic Media

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Summary

The elastic constants that control *P*- and *SV-wave* propagation in a transversely isotropic media can be estimated by using *P*- and *SV-wave* traveltimes from either cross-well or VSP geometries. The procedure consists of two steps. First, elliptical velocity models are used to fit the traveltimes near one axis. The result is four elliptical parameters that represent direct and normal moveout velocities near the chosen axis for *P*- and *SV-waves.* Second, the elliptical parameters are used to solve a system of four equations and four unknown elastic constants. The system of equations is solved analytically yielding simple expressions for the elastic constants as a function of direct and normal moveout velocities. For S&waves the estimation of the corresponding elastic constants is easier because the phase velocity is already elliptical. The procedure, introduced for homogeneous media, is generalized to heterogeneous media by using tomographic techniques. The technique is illustrated with synthetic and field data examples from cross-well geometries.

Introduction

The effect of velocity anisotropy on wave propagation in homogeneous and heterogeneous media has been the subject of numerous publications. Careful forward modeling has helped interpreters to understand how velocity anisotropy manifests itself in field data. Few attempts have been made, however, to estimate the parameters that describe the complexity of velocity anisotropy, namely, the elastic constants. Estimation of the elastic constants is important because it can aid in lithologic discrimination and fracture orientation, reveal anisotropic properties of the medium not obvious in the data, and provide further imaging or full waveform inversion algorithms with background models that can be refined iteratively.

Previous studies that have estimated variations of anisotropy with position have used "intermediate" models that make simplifications about both anisotropy and heterogeneity. The selection of the model is based on two factors: the prior information available about the medium and the geometry used to record the data. When selecting the model for anisotropy, these two factors make transverse isotropy (TI) a good candidate because, in the one hand, TI is a very common form of anisotropy in the subsurface and, on the other hand, the 3-D multicomponent information that is necessary to study more complex symmetries is not usually recorded. For analogous reasons, layered models (1-D) have been used routinely to describe the heterogeneities. Therefore, not surprisingly, several authors (Hake et al., 1984; Byun and Corrigan, 1990; Sena, 1991) have chosen the combination l-D/T1 to describe their models when estimating elastic constants.

Unfortunately, all the preceding methods fail when the data are not wide aperture, which is often the case with VSP and cross-well experiments.

We show in this paper how to obtain the elastic constants that control *P*- and *SV-wave* propagation in TI media from limited aperture traveltimes, either from VSP or from cross-well geometries. We start by fitting the traveltimes for *P*- and *SV-waves* with elliptical time-distance relations near a single axis (either vertical or horizontal). The result is four velocities: two based on the time-of-arrival and distance along a symmetry axis (the direct velocities) and two based on the differential traveltime and differential distance as the direction is perturbed (the normal moveout velocities). These four elliptical parameters are used to solve analytically a system of four equations and four unknown elastic constants. Since the procedure is based on fitting the data with elliptical velocity models, it is exact only when estimating elastic constants from *SH-wave* traveltimes.

The recording aperture is constrained in two different ways. First, it should not be too small to ensure that there is enough curvature to estimate the normal moveout velocities. Second, it should not be too wide to ensure that the elliptical fit remains accurate for the given wave type.

The calculations presented here are valid for homogeneous media. When the model is heterogeneous, it can be described as a superposition of homogeneous regions, and the elliptical parameters needed at each region are estimated tomographically, as explained by Michelena et al. (1993) and Michelena (1992). The result is 2-D images of elastic constants.

The equations we use in this paper to transform elastic constants into elliptical parameters (forward mapping) are not new. They are the same as the ones summarized by Muir (1990), which can also be found in Levin (1979) and Levin (1980). What is new is the simultaneous solution of these equations near each axis to obtain elastic constants as a function of elliptical parameters (inverse mapping).

We start by rederiving the basic equations of the forward mapping from the expression of *P*- and *SV-wave* phase velocities in TI media. The calculations are done near the horizontal axis. Then, using these expressions, we solve the inverse mapping analytically. The final section illustrates the use of the technique when estimating the elastic constants of homogeneous and heterogeneous media from traveltimes measured around the horizontal.

From elastic constants to phase velocities

The phase velocity expression for *P*- and *SV-waves* in TI media is (Auld, 1990)

$$
\begin{aligned}
2W_{P,SV}(\theta) &= (W_{33}+W_{44})\cos^2\theta + (W_{11}+W_{44})\sin^2\theta \\
&\pm \left([(W_{33}-W_{44})\cos^2\theta - (W_{11}-W_{44})\sin^2\theta]^2\right. \\
&+ \left. 4(W_{13}+W_{44})^2\sin^2\theta\cos^2\theta\right)^{1/2}, \qquad (1a)
\end{aligned}
$$

where $W_{P,SV}(\theta)$ is the phase velocity squared and θ is the phase angle from the vertical. W_{ij} is the $(ij)^{th}$ elastic modulus divided by density, with units of velocity squared; we refer to the quantity W_{ij} as an "elastic constant" in the remainder of the paper. The plus sign $(+)$ in front of the square root corresponds to P-waves and the minus sign $(-)$ to SV-waves. For SH-waves, the expression for the phase velocity is (Auld, 1990)

$$W_{SH}(\theta) = W_{44}\cos^2 e + W_{66}\sin^2\theta. \quad (1b)$$

Expanding equation (la) around $\theta = 90$ and neglecting terms in $\cos^4\theta$, results in:

$$\begin{aligned}2W_{P,SV}(\theta) &= (W_{11} + W_{44})\sin^2 e + (W_{33} + W_{44})\cos^2 e\\ &\pm [(W_{11} - W_{44})\sin^2 e - (W_{33} - W_{44})\cos^2 e\\ &+ \frac{2(W_{13} + W_{44})^2}{W_{11} - W_{44}}\cos^2\theta]. \quad (2)\end{aligned}$$

Choosing the positive root yields the P-wave phase velocity near the horizontal axis, as follows:

$$W_P(\theta) = W_{P,x}\, s^2 + W_{P,znmo}\, c^2, \quad (3)$$

where $c = \cos\theta$, $s = \sin\theta$,

$$W_{P,x} = W_{11}, \quad (4)$$

and

$$W_{P,znmo} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}}. \quad (5)$$

$W_{P,x}$ is the horizontal P-wave phase velocity squared and $W_{P,znmo}$ is the vertical normal moveout (NMO) phase velocity squared. Choosing the negative root in equation (2) yields SV-wave phase velocities near the horizontal axis, as follows:

$$W_{SV}(\theta) = W_{SV,x}\, s^2 + W_{SV,znmo}\, c^2, \quad (6)$$

where

$$W_{SV,x} = W_{44}, \quad (7)$$

and

$$W_{SV,znmo} = W_{33} - \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}}. \quad (8)$$

The expression for SH-wave phase velocities near the horizontal axis is

$$W_{SH}(\theta) = W_{SH,x}\, s^2 + W_{SH,znmo}\, c^2, \quad (9)$$

where

$$W_{SH,x} = W_{66}, \quad (10)$$

and

$$W_{SH,znmo} = W_{SH,z} = W_{44}. \quad (11)$$

In the rest of the paper we refer to the elliptical parameters $W_{P,x}$, $W_{P,znmo}$, $W_{SV,x}$, $W_{SV,znmo}$, $W_{SH,x}$, $W_{SH,z}$, and $W_{SH,znmo}$ as W_*, direct or NMO phase velocity squared for P-, SV-, and SH-waves. The corresponding equations for near vertical propagation can be obtained by interchanging x and z, $\cos\theta$ and $\sin\theta$, and W_{11} and W_{33} in equations (2) to (11).

From phase velocities to elastic constants

When the phase angle (measured from the vertical) is close to 90 degrees, the expressions for the elastic constants as a function of P- and SV-wave phase velocities are obtained by solving the system of equations (4), (5), (7), and (8), with the following result:

$$W_{11} = W_{P,x}, \quad (12a)$$

$$W_{44} = W_{SV,x}, \quad (12b)$$

$$W_{13} = \sqrt{(W_{P,znmo} - W_{SV,x})(W_{P,x} - W_{SV,x})} - W_{SV,x}, \quad (12c)$$

$$W_{33} = W_{SV,znmo} + W_{P,znmo} - W_{SV,x}. \quad (12d)$$

The estimation of W_{33} from near-horizontal phase velocities [equation (12d)] is the sum of NMO velocities minus W_{44}. Michelena (1993) shows that when estimating velocities tomographically, NMO velocities correspond to the smallest singular values of the problem. The largest singular values correspond to velocities estimated from rays that travel along the axes. Therefore, as expected, estimating W_{33} from cross-well traveltimes alone is a harder problem than estimating W_{11} from the same data. The opposite is true when estimating W_{33} and W_{11} from VSP measurements.

From traveltimes to phase velocities

Equations (3) and (6) show that the phase velocities of P- and SV-wave are elliptical near the axes of symmetry. Those of S&waves are also elliptical [equation (lb)]. When the phase velocity has an elliptical shape, the corresponding impulse response is also elliptical (Levin, 1978; Byun, 1982). Therefore, the group slowness expression that corresponds to these equations has the general form

$$S^2(\phi) = S_z^2\cos^2\phi + S_x^2\sin^2\phi, \quad (13)$$

where ϕ is the ray angle measured from the vertical and S_* (the ray slowness) is

$$S_*^2 = \frac{1}{W_*}. \quad (14)$$

To estimate S,, we use the expression for the traveltime of a ray that travels a distance $l = \sqrt{\Delta x^2 + \Delta z^2}$ between two points:

$$t^2 = \Delta x^2 S_x^2 + \Delta z^2 S_z^2. \quad (15)$$

This equation, which has the same form as the isotropic moveout equation, is obtained after multiplying equation (13) by 1^2.

From traveltimes to elastic constants

The procedure to estimate the elastic constants of a homogeneous TI medium from traveltime measurements near one axis of symmetry is the following:

1. Fit the traveltimes with elliptically anisotropic models, one model for each wave type. This gives direct and NMO group slownesses.

2. From the direct and NM0 group slownesses, find the corresponding direct and NM0 phase velocities, using equation (14).

3. From the estimated phase velocities, find the elastic constants using the equation (12) (for cross-well geometries). In the case of *SH* waves, the estimated phase velocities squared are the same as the corresponding elastic constants.

We generalized this procedure to heterogeneous media by describing the model as a superposition of homogeneous blocks. The elliptical velocitities are estimated tomographically as explain in Michelena et al. (1993) and Michelena (1992).

Synthetic example in homogeneous medium

Figure 1 shows the result of applying the procedure described in the preceding section to estimate the elastic constants that control *P*- and *SV-wave* propagation in a homogeneous TI medium. The impulse response is sampled at four different angles near the horizontal. The elastic constants that describe the medium are: $\sqrt{W_{11}}$ = 2256, $\sqrt{W_{33}}$ = 1919, $\sqrt{W_{13}}$ = 1699, and $\sqrt{W_{44}}$ = 658, all with units of (ft/s). The agreement between given and estimated impulse responses is excellent and the error in the estimation of the elastic constants is $\approx$ 2% for W_{13} (F) and $\approx$ 1% for W_{33} (C).

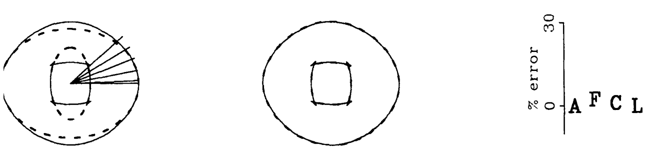

Figure 1: Left: impulse response for *P*- and *SV-waves* (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). All ray angles shown are used simultaneously to calculate the elliptical approximations. Center: given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). Right: absolute value of the error made in the estimation of the elastic constants. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}.

Synthetic example in heterogeneous medium

Synthetic traveltimes for *P*- and *SV-waves* were generated through the layered TI model shown in Figure 2 (continuous lines). The ray tracing algorithm used to compute these traveltimes is described in Michelena (1993). In order to obtain the elliptical velocities that equation (12) requires, the synthetic data were inverted tomographically as explained by Michelena et al. (1993) and Michelena (1992). Only ray angles between 0 and 30 degrees were used for the inversion. Figure 2 (dashed lines) shows the result of transforming the estimated elliptical velocities at each layer into elastic constants. The agreement between given and estimated elastic constants is very good.

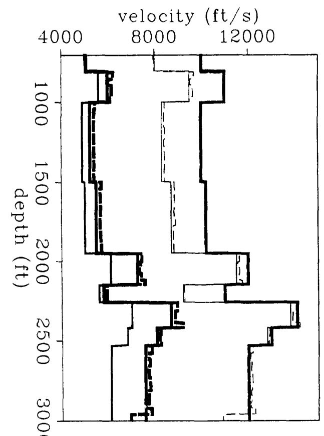

Figure 2: Elastic constants that control *P*- and *SV-wave* propagation. Continuous lines: given. Dashed lines: estimated. From left to right the four pairs of curves represent V_{44}, V_{13}, V_{33}, and V_{11}, respectively. V_{ij} is the squared root of W_{ij}. The density is assumed to be unity.

Cross-well field data example

Cross-well data were recorded at a carbonate reservoir of the Permian Basin in west Texas. This field has large oil reserves. It was discovered in 1926 and has been under continuous water-flooding since the 1960's. The field produces mainly from intertidal and shallow-shelf dolostones and siltstones of the Grayburg formation, which form a stratigraphic-structural trap. Reservoir performance has been stimulated by hydraulic fracturing.

A cylindrical piezoelectric bender was used as the source, a linear upsweep from 250 to 2000 Hz. Well spacing is 184 feet. The receiver system was a nine-level array of hydrophones. Source and receiver vertical spacing was 2.5 feet, from 1650 to 2150 feet.' The survey consists of nearly 36000 traces (201 sources x 178 receivers) sampled at 0.2 ms. A typical common receiver receiver gather is shown in Figure 3. More details about the data acquisition can be found in Harris et al. (1992).

Figure 4 shows the elastic constants estimated for the field site when applying the technique previously described. V_{11} and V_{33} (horizontal and vertical P-wave velocities, respectively) vary more rapidly than V_{44} (*SV-wave* velocity). The difference $(V_{11} - V_{33})$ alternates between zero or negative in the interval between 1700 and 2100 feet. If we assume that the anisotropy is caused by fine layering, these changes can be explained by a sequence of isotropic strata and anisotropic strata with horizontal axes of symmetry, probably vertically fractured.

Conclusions

We have shown how to estimate the elastic constants of homogeneous TI media from *P*-, *SV*-, and *SH-wave* traveltimes near a single axis of symmetry (either from VSP or cross-well geometries). The technique uses the parameters

[1] Field location and reservoir depths are changed for purposes of presentation in this paper.

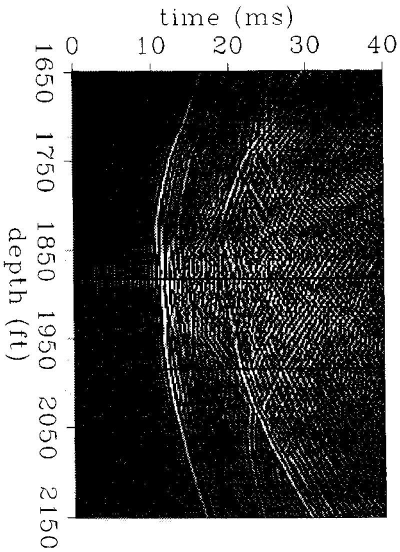

Figure 3: Common receiver gather recorded at 1880 feet. The source depth interval is 2.5 feet. The well-to-well separation is 184 feet. First arriving compressional and shear waves are clearly visible at most vertical offsets. The target of the experiment is a reservoir between 1850 and 1960 feet.

obtained by fitting traveltimes near one axis with elliptical models. For SH-wave traveltimes, the estimation of the corresponding TI elastic constants is trivial because SH-wave phase velocities are also elliptical in TI media. For P- and SV-wave traveltimes, four parameters are needed to estimate the phase velocities at all angles from measurements near one axis. These parameters are the direct and NMO phase velocities for P- and SV-waves, which can be estimated tomographically when the medium is heterogeneous. The transformation from elliptical parameters to elastic constants is simple.

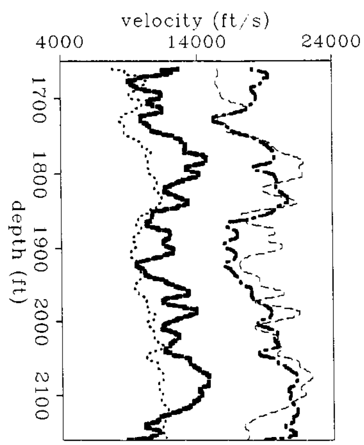

Figure 4: Elastic constants (in units of velocity) estimated at the site assuming a TI medium. Dotted line: V_{44}. Thick-dashed line: V_{13}. Dotted-dashed line: V_{11}. Thin-dashed line: V_{33}. Reservoir between 1850 and 1960 feet.

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