Stratigraphic inversion of poststack PS converted waves data

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Summary

We show in this paper that conventional, poststack stratigraphic inversion of PS converted waves based on the convolutional model for the seismic trace yields impedance estimates, pseudo S-wave impedances, that are the product of a pseudo density by S-wave velocity. This pseudo density transforms the equation for the PS reflection coefficient for near vertical incidence into an expression that is equivalent to the well-known expression of the normal incidence reflection coefficient for P-waves. The pseudo density is a function of the medium density and the V_P/V_S ratio. The dependence of the angle in the PS reflection coefficient is eliminated when stacking and, therefore, near offset stacked PS traces can be modeled and inverted assuming the convolutional model for the seismic trace. We also derive a simple expression to estimate medium density that depends on $V_P V_S$, V_P/V_S, P-wave impedance, and pseudo S-wave impedance estimated all from conventional velocity analysis and stratigraphic inversion of near offset PP and PS data.

Introduction

The estimation of medium properties from changes in P-wave reflectivity has been performed successfully during many years using not only poststack data when we want to estimate changes in P-wave impedance (Russell and Hampson, 1991), but also prestack data when we want to estimate changes in S-wave velocities and densities that affect the reflectivity at large offsets (Demirbag et al., 1993).

The estimation of changes in medium properties from PS converted waves, however, is commonly performed using only prestack data (Stewart, 1991). The reason for not using poststack PS converted waves to estimate changes in medium properties is because we have not been able to find a model for PS stacked data that has the simplicity of the convolutional model we assume to model PP stacked data. Unlike PP reflectivity, PS reflectivity is zero for normal incidence and, therefore, the convolution of the normal incidence reflectivity series with a wavelet is meaningless as a way to model PS stacked data.

We show in this paper that near offset PS stacked data can be approximated by the convolution of the near normal incidence PS reflectivity $[R_{PS}(\theta \approx 0)]$ with a wavelet. Even though the PS reflectivity is zero for normal incidence, we demonstrate that the stacked trace we assign to the common conversion point location is proportional to changes in a quantity that we defined as a pseudo S-wave impedance. This quantity is obtained after replacing the density term in the approximation of the PS reflection coefficient $R_{PS}(\theta \approx 0)$ by a new pseudo density. This new density transforms $R_{PS}(\theta \approx 0)$ into an expression that has the same functional form as the P-wave normal incidence reflection coefficient. We also derive an expression for the density of the medium that depends on the product and ratio of interval V_P and V_S velocities and the product of the pseudo S-wave impedance and the P-wave impedance. A simple synthetic example demonstrates the validity of the new approximations.

Near offset PS forward modeling

According to Aki and Richards (1980), the expression for the PS-wave reflection coefficient on a solid-solid interface between medium 1 and medium 2 is

$$R_{PS} = -\frac{V_P}{2V_S}\tan\psi[A\frac{\Delta\rho}{\rho} - B\frac{\Delta V_S}{V_S}], \qquad (1)$$

where

$$A = 1 - 2\frac{V_S^2}{V_P^2}\sin^2\theta + 2\frac{V_S}{V_P}\cos\theta\cos\psi,$$

and

$$B = 4\frac{V_S^2}{V_P^2}\sin^2\theta - \frac{4V_S}{V_P}\cos\theta\cos\psi.$$

The angle θ is the incidence angle of the P-wave and ψ is the emergence angle of the PS converted wave. This equation is valid when $\Delta V_P, \Delta V_S$, and $\Delta\rho$ are all small.

For small angle of incidence θ, we can approximate equation 1 of the PS-wave reflection coefficient as

$$R_{PS} \approx -2\sin\psi\left[\left(\frac{1}{4}\frac{V_P}{V_S} + \frac{1}{2}\right)\frac{\Delta\rho}{\rho} + \frac{\Delta V_S}{V_S}\right]. \qquad (2)$$

As we can see, for positive contrasts of medium properties across the interface, the PS-wave reflection coefficient is negative for near vertical rays, unlike the reflection coefficient for normally incident P-waves which is always positive for positive contrasts of medium properties

Similarly to near offset P-wave traces, near offset PS-wave traces can be approximated as the convolution of PS-wave reflectivity series

$$r_{PS}(t) = \sum_i^N R_{PSi}(\psi_i)\delta(t - \tau_{PSi})$$

with a wavelet W_{PS} as follows:

$$t_{PS}(t) \approx r_{PS}(t) * W_{PS}(t), \qquad (3)$$

where N is the total number of interfaces and τ_{PSi} indicates the position in time of the i^{th} interface.

The PS-wave stacked trace T_{PS} is obtained after integrating equation 3 from zero to the converted wave angle ψ_m that corresponds to the farthest trace in the NMO corrected, common conversion point gather. The result is

$$T_{PS}(t) \approx 2(\cos\psi_m - 1)\sum_i^N \left[\left(\frac{1}{4}\frac{V_{Pi}}{V_{Si}} + \frac{1}{2}\right)\frac{\Delta\rho_i}{\rho_i} + \frac{\Delta V_{Si}}{V_{Si}}\right]\delta(t-\tau_{PSi}) * W_{PS}(t), \qquad (4)$$

which means that, after assigning all scale factors to a new wavelet $\widehat{W}_{PS}$, the stacked trace of the horizontal component of the geophone can be approximated by

$$T_{PS}(t) \approx \sum_i^N \left[\left(\frac{1}{4}\frac{V_{Pi}}{V_{Si}} + \frac{1}{2}\right)\frac{\Delta\rho_i}{\rho_i} + \frac{\Delta V_{Si}}{V_{Si}}\right]\widehat{W}_{PS}(t-\tau_{PSi}). \qquad (5)$$

As we can see, the stack of the horizontal component of the geophone (equation 5) produces an average trace that contains information about changes in medium properties, which is also the case for the stack of near offset vertical component traces. However, the main difference between these two cases is that for P-waves the stack is also proportional to the reflection coefficient at zero offset whereas for PS-waves it is not, since the PS-wave reflection coefficient is zero for normal incidence. As Stewart et al. (1998) point out, stacked PS sections represent an average of the amplitude versus offset response across the set of offsets that enter into the common conversion point gather.

There are many commercial software packages available today to estimate P-wave impedance from PP data. However, if we want to use the same tools with PS data, we need to transform the equation of the PS reflection coefficient in such a way it has the same functional form as the PP reflection coefficient. Next section explains this change in detail.

Estimation of pseudo S-wave impedance

The equations for the near vertically incident PP- and PS-waves respectively have similar dependence on both velocity and density changes. These equations, however, are not functionally identical because the factor that multiplies the changes in density differs from one equation to the other. If we want them to be equivalent, we need to find a quantity $\widehat{\rho}$ proportional to density such that

$$\frac{\Delta\widehat{\rho}}{\widehat{\rho}} = \left(\frac{1}{4}\frac{V_P}{V_S} + \frac{1}{2}\right)\frac{\Delta\rho}{\rho}. \qquad (6)$$

After assuming small changes in density across the interface, the integration of equation 6 yields the pseudo density $\widehat{\rho}$ we were looking for (which has no units of density):

$$\widehat{\rho} = \rho^{\left(\frac{1}{4}\frac{V_P}{V_S}+\frac{1}{2}\right)}. \qquad (7)$$

After replacing ρ from equation 7 into equation 2 (remembering that $\Delta\rho/\rho \approx \Delta\log\rho$), we obtain

$$R_{PS} \approx -2\sin\psi\left[\frac{\Delta\widehat{\rho}}{\widehat{\rho}} + \frac{\Delta V_S}{V_S}\right], \qquad (8)$$

which is an expression equivalent to the normal incidence reflection coefficient for P-waves except for the term $\sin\psi$ that factors out after stacking. As shown in Sheriff and Geldart (1982), this expression can be also expressed as

$$R_{PS} \approx -2\sin\psi\frac{\widehat{Z}_{S2} - \widehat{Z}_{S1}}{\widehat{Z}_{S2} + \widehat{Z}_{S1}}, \qquad (9)$$

where $\widehat{Z}_{Sj} = \widehat{\rho}_j V_{Sj}$ is the pseudo S-wave impedance. Therefore, if we perform conventional stratigraphic inversion (Russell and Hampson, 1991) of PS stacked data, we obtain estimates of $\widehat{Z}_S$. Only for the cases when the density does not change vertically or when $V_P/V_S = 2$, the pseudo S-wave impedance coincides with the actual S-wave impedance.

Estimation of density

After estimating P-wave impedance and pseudo S-wave impedance from stratigraphic inversion of PP and PS data respectively, we can easily show that the medium density can be estimated from near offset data by using the expression

$$\rho = \left(\frac{Z_P\widehat{Z}_S}{V_P V_S}\right)^{\frac{1}{\alpha}}, \qquad (10)$$

where

$$\alpha = \frac{1}{4}\frac{V_P}{V_S} + \frac{3}{2}. \qquad (11)$$

The density estimate we obtain from equation 10 is consistent with both PP and PS data. However, it requires independent estimates of $V_P V_S$ and V_P/V_S that can be obtained either from a dipole sonic log or from interval velocities obtained after conventional velocity analysis of PP and PS data. The ratio V_P/V_S can be estimated also from interval traveltimes by using the well known formula

$$\frac{V_P}{V_S} = 2\frac{\Delta t_{PS}}{\Delta t_P} - 1, \qquad (12)$$

where Δt_{PS} and Δt_P are the traveltime differences between top and bottom of the interval of interest. Equation 10 is obtained after multiplying $Z_P = \rho V_P$ by $\widehat{Z}_S = \hat{\rho} V_S$. Other expressions for ρ can be obtained by combining Z_P and $\widehat{Z}_S$ in different ways.

Synthetic example

To test the ideas presented in previous sections, we generated synthetic PP and PS reflectivities using the exact expressions of the Zoeppritz equations (Aki and Richards, 1980). To model the synthetic data, we used real dipole sonic log velocities and densities from a well located in eastern Venezuela.

Figures 1 and 2 show the PP and PS reflectivities calculated for angles between 1 and 30 degrees. The purpose of this modeling was to simulate the range of angles that enter into a typical stacked trace for PP and PS seismic records. After stacking the reflectivities for all angles, we used the recursive formula (Russell and Hampson, 1991)

$$Z_{j+1} = Z_j \frac{1+R_j}{1-R_j} \tag{13}$$

to estimate the corresponding impedances Z_P and $\hat{Z}_S$. In order to use equation 13, we needed to be careful about selecting the proper scaling of the reflection coefficients. Figures 3 and 4 show the results of the recursive inversions, which are P-wave and pseudo S-wave impedances. We observe that real and estimated impedances (solid and dashed curves respectively) are in close agreement with each other, which confirms that stratigraphic inversion of PS records yields the pseudo S-wave impedances introduced in equation 9.

Finally, we used the estimated P-wave and pseudo S-wave impedances to estimate the density of the medium. Figure 5 shows that the densities estimated using equation 10 reproduce very well the real ones. After some experimentation, we found that this formula is not too sensitive to variations in V_P/V_S, and good density estimates can be obtained using only average V_P/V_S values.

Conclusions

We have demonstrated that poststack PS converted waves can be inverted for pseudo impedances that are the product of S-wave wave velocities and a pseudo density that equals the medium density for the case $V_P/V_S = 2$. For other values of V_P/V_S, this pseudo density transforms the equation of the PS reflection coefficient for small angle of incidence into an equation that is functionally identical, except for an angle factor, to the well known expression of the PP normal incidence reflection coefficient.

Impedances derived from poststack inversion of PP and PS data can be combined in various ways into a single expression to estimate the density of the medium. The particular expression for the density proposed in this paper depends, besides impedances, on the product and the ratio of P- and S-wave velocities.

We have tested the validity of the new approximations with synthetic data generated using real velocities, real densities, and the exact expressions of the Zoeppritz equations. However, more research needs to be done to understand the advantages and difficulties of using this formulation to invert real, poststack converted waves data.

Acknowledgments

We thank PDVSA Intevep for permission to publish this paper. We also thank Ezequiel Gonzalez for his important suggestions.

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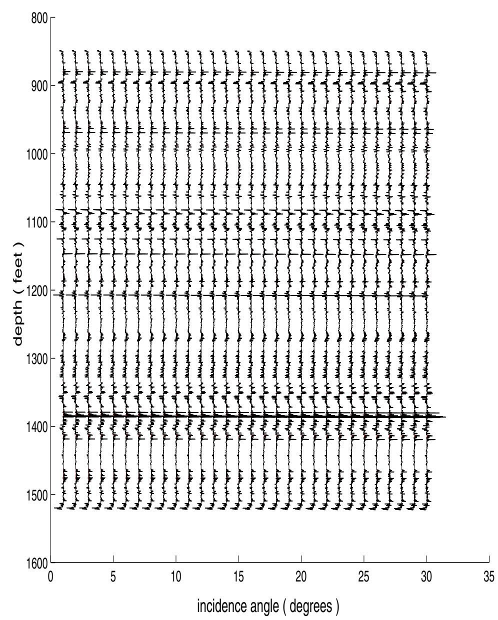

Fig. 1: Synthetic P-wave reflectivity R_{PP} for angles between 1 and 30 degrees.

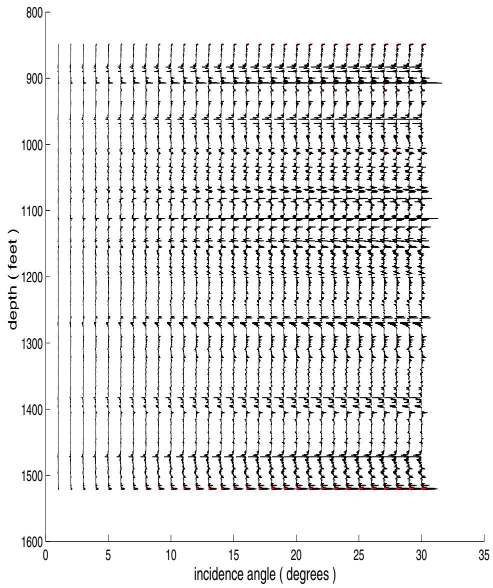

Fig. 2: Synthetic PS-wave reflectivity R_{PS} for angles between 1 and 30 degrees.

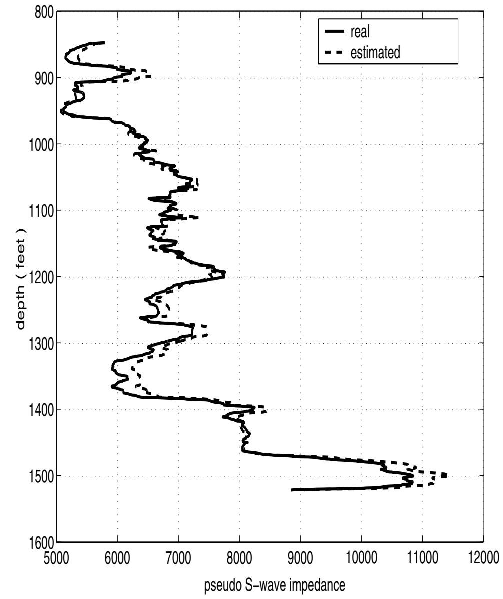

Fig. 4: Comparison between real and estimated pseudo S-wave impedances.

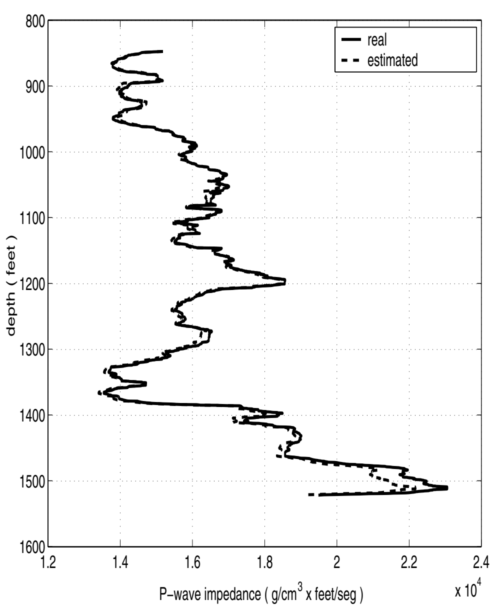

Fig. 3: Comparison between real and estimated P-wave impedances.

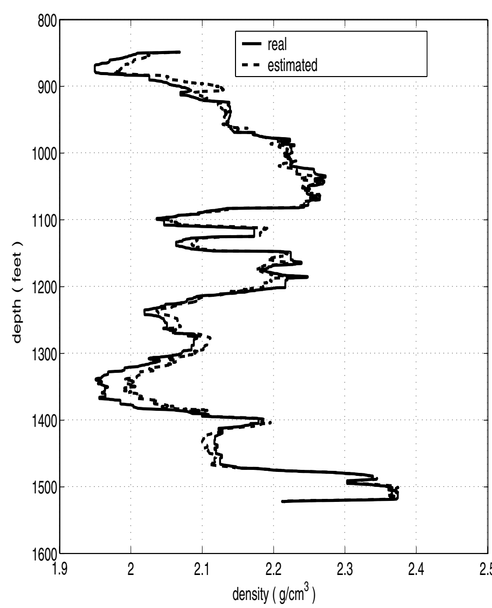

Fig. 5: Comparison between real and estimated densities using equation 10.