Lithologic Characterization of a Reservoir Using Continuous-Wavelet Transforms

Giselle Álvarez, Bruno Sansó, Reinaldo J. Michelena, and Juan Ramón Jiménez

***Abstract*—We consider the problem of characterizing the lithology of a reservoir using gamma ray logs as well as seismic traces around the well. We first calculate the continuous-wavelet transform of the the data and then use the fact that the energy of such transformation is proportional to a power of its scale. The technique consists in estimating the power transformation obtaining a set of values of the same size as the original data and then modeling the distribution of these values using a double exponential. We find that wells that are predominantly sand correspond to distributions that are significantly different from those that correspond to wells that are predominantly gravel. This happens in both cases: gamma rays and seismic traces. We use this characterization to classify other points in the reservoir.**

***Index Terms*—Bayesian statistical methods, continuous-wavelet transform, lithology classification.**

I. Introduction

OIL DOES NOT accumulate in all types of rock. Once generated at the source rock, hydrocarbons may travel large distances throughout the porous medium until they find the proper conditions that help to trap them. First, they need to find a porous rock (the reservoir rock) where they can rest until we reach them; second, they need to find a seal that prevents them from traveling any further. Typical examples of reservoir and seal rocks are sandstones and shales, respectively. For this reason, determining rock types in a certain oil exploration area becomes a fundamental task that helps to exploit the existing reserves more efficiently. At well locations, we can record various types of well logs that can help us differentiate among rock types. Since this information is restricted to a few feet around the well location, the classification of rock types across the reservoir usually relies on crude interpolations of log measurements from well locations to the interwell space. However, when seismic data are available, we can use them to guide such interpolations, or we can analyze how the seismic data themselves respond to changes across different rock types that have been previously labeled at the well.

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Various authors have proposed methods to classify rock types by analyzing subtle changes in seismic waveform in the interwell space [1]–[4]. Most of these methods rely on some measure of change in the shape of the waveforms in the seismic data or other attributes derived from them.

We show in this paper a method to analyze the shape of the seismic waveforms based on their fractal behavior. Change in such behavior, as shown in [5], can be related to change in rock type. The method we propose is based a statistical framework and provides a probabilistic assessment of the uncertainties. It is beyond the scope of this work to compare our proposal with other methods for the classification of rock type using seismic data. Other authors [6]–[10] have presented neural-net-based statistical methods applied to geophysical inverse problems for estimation of rock properties. In particular, a Caianiello neural network has been suggested. This consists of neural wavelet estimation, input signals reconstruction, and nonlinear factor optimization. These algorithms solve a wide range of geophysical inverse problems.

Rocks exhibit fractal behavior [11] that tells whether there is order and structure in the internal relation among their different scales. Measurements of properties of rocks at these different scales may also show such a behavior even though the relations between such properties and the rocks themselves may be rather complicated. Todoeschuck *et al.* [12] show that the acoustic reflectivity of a sequence of rocks is also fractal, which suggests that quantities derived from properties that show fractal behavior may also be fractal. Jiménez *et al.* [13] show that changes in the fractalility of acoustic reflectivity may be related to changes in the predominant rock type in the area where the log was recorded. Moreover, they show that seismic traces (that can be modeled as the convolution of the reflectivity series with a wavelet) show distinctive fractal behaviors for different lithologies. They used this observation to classify rock types across a reservoir using three-dimensional seismic data.

The method proposed in [13] is based on the estimation of fractal-related properties of the signal using a discrete wavelet transform. Using resistivity logs from two different wells located in areas that penetrate sandy and shaley environments respectively, the authors perform a discrete wavelet decomposition and compute the variance of the coefficients at every level of detail; then they plot the result against its corresponding scale. They observed that the slopes of the resulting straight lines, which provide estimates of the fractal dimension of the signals, are related to the dominant lithology around the wells. This work is the main motivation of our paper. We are interested in looking at the problem of the characterization of the lithology from an inferential point of view and measure the uncertainty in the char-

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acterization, which is not considered in [13]. The starting point of our analysis is the assumption of a probability model that involves unknown parameters to be estimated.

Using a discrete wavelet transform, we are able to obtain a single estimate of the fractal dimension of each signal. Continuous-wavelet transforms (CWTs) are not subject to decimation and can thus produce a numerous assembly of estimates that can be used to infer about a probability distribution for the fractal dimension. This is the starting point of the present paper. Then, we observe that the distribution of the power of the scale of the wavelet transform corresponds to a double-exponential distribution, which depends on a location and a scale parameter. We consider a statistical test to assess that the location parameter is different for seismic traces recorded in different types of rocks. The observation that the data can be fit accurately with a double-exponential probability distribution, whose parameters depend on the rock type where the signal is recorded, is the essential tool for the proposed classification method of our paper. Our test is based on a Bayesian approach, which allows us to use probabilities to quantify uncertainties. Once we observe that the fractal dimensions of different lithologies are statistically significantly different, we can use the double-exponential model to classify signals, with the added bonus that we quantify how uncertain the classification is. A map is drawn using the probabilities for each type of lithology in the reservoir.

The paper is organized as follows. In Section II, we present the definition of CWTs and a property of them that is essential for the development of this work. Section III describes the data used in this study. In Section IV, we consider the problem of the characterization of the lithology, estimating the power of the scale of the wavelet transformation. Section V formalizes the comparison between the proposed distribution in the previous section, following a Bayesian approach. Finally, in Section VI, we describe the classification procedure that consists in calculating the posterior probability that a new observation belongs to one of the two distributions proposed for the power parameter.

II. CWT

Let ψ be a complex-valued square-integrable function that satisfies the *admissibility condition*

$$C_\psi = \int_0^\infty \frac{|\psi(x)|^2}{|\omega|} d\omega \leq \infty.$$

Then ψ is called a *wavelet function*. Define $\psi_{a,b}(x)$ as the family of functions given by translations and rescales of $\psi(x)$

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right).$$

For any square-integrable function f, the *CWT* is defined as a function of two variables

$$\mathrm{CWT}_f(a,b) = \int f(x)\overline{\psi_{a,b}(x)}dx$$

where $\overline{\psi_{a,b}(x)}$ denotes the conjugate of $\psi_{a,b}(x)$. The dilation and translation parameters a and b vary continuously over $\mathbb{R}^+ \times \mathbb{R}$, respectively. When the admissibility condition is satisfied, it is possible to find the inverse continuous transformation

$$f(x) = \frac{1}{C_\psi} \int_0^\infty \int_{-\infty}^\infty |\mathrm{CWT}_f(a,b)|^2 \frac{1}{a^2} db\, da.$$

We consider a discretization of the CWT. The CWT of a univariate function is a function of two variables. However, to minimize the redundancy in the transformation, one selects discrete values a and b to produce an invertible transformation, but any sampling that preserves all information about the decomposed function cannot be coarser than the critical sampling, which is defined by $a = 2^{-j}$ and $b = k2^{-j}$, $j, k \in \mathbb{Z}$, which produce a minimal basis. Any coarser sampling will not give a unique inverse transformation. For more details see [14].

An important property of CWT_f, which is key for the development of the present paper, is that there is a link between the regularity of f and that of the energy of CWT_f. In fact, if $f \in \mathbb{C}^\alpha$ (Hölder space with exponent α) then we have that

$$(\mathrm{CWT}_f(a,b))^2 \propto a^{2\alpha+1}. \tag{1}$$

For details see [15]. Fixing b, we can use (1) to estimate α by regressing $\log_2([\mathrm{CWT}_u(a,b)]^2)$ on $\log_2(a)$. This idea is developed in [16] to characterize properties of turbulent flows in Duke Forest, NC.

It is known that, based on well-log measurements, a number of geophysically important rock properties like porosity, resistivity, density, to name a few, have been found to exhibit multifractal behavior [17]–[20], meaning that the Hölder exponent α takes different values in time. Such a structure, as argued in [16], calls for using the CWT, given its ability to estimate local scaling exponents. It has the advantage over the Fourier transform of giving information not only of the global frequency content of a signal but also shows where, in time, certain frequency components occur.

In this paper, we present a method to characterize both the seismic data around each well and γ-ray logs based on property (1). We estimate the power α for each depth b, obtaining a set of values of the same size as the original signal; we then model the distribution of these values using a double exponential. We find that different lithologies are associated with different values of the location parameters of such distributions.

III. Description of the Data

For the purpose of this study, we analyze a dataset consisting of 17 γ-ray logs and 100 km of two-dimensional seismic data recorded in western Venezuela, in the Barinas-Apure basin. The wells penetrated rock columns whose lithologies were either predominantly sands or predominantly gravel. We also have three additional γ-ray logs from wells located in transition zones between sand and shale.

Even though seismic traces are proportional to changes in medium properties (reflectivity) rather than the properties themselves, we still used seismic traces to perform our analysis. However, we believe that for the purpose of these analyses, the acoustic impedance obtained from the seismic trace may be more adequate, since it is related to interval medium properties.

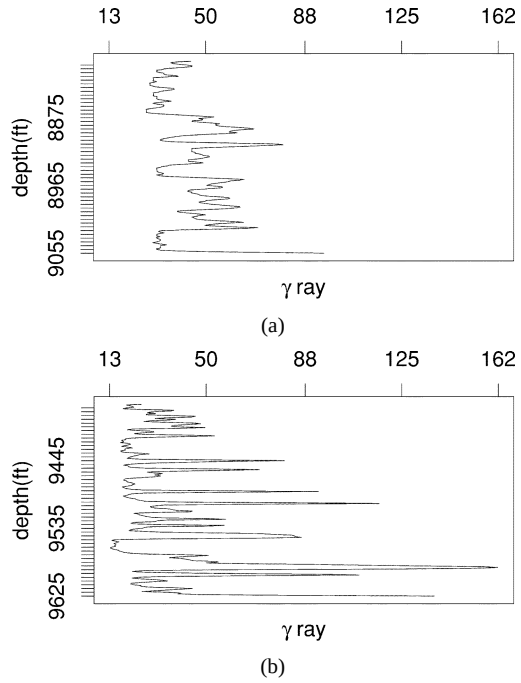

Fig. 1. γ-ray logs for (a) gravel well and (b) sand well.

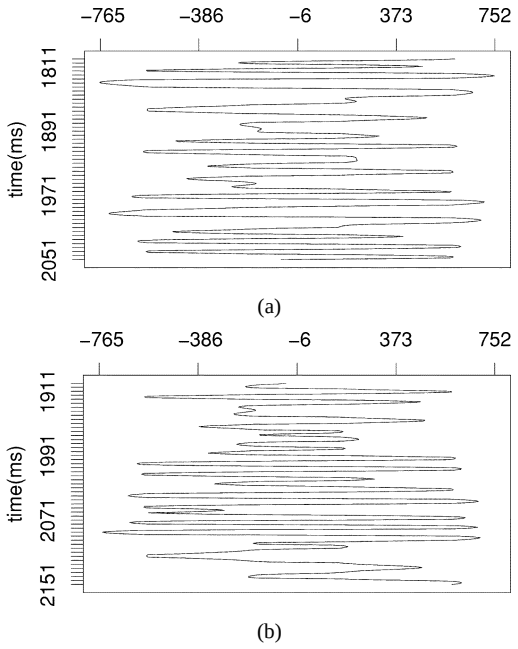

Fig. 2. Seismic traces for (a) gravel well and (b) sand well.

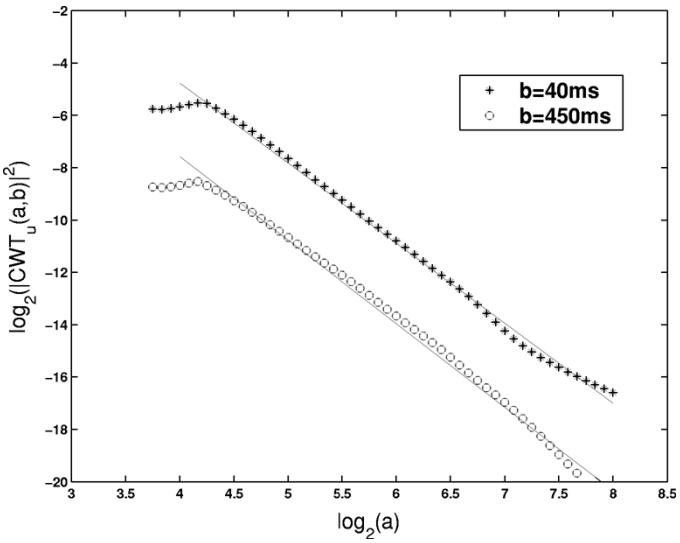

Fig. 3. Estimated scaling exponent α for $b = 40$ ms and $b = 450$ ms is given by the slopes of the straight lines.

The choice of the window length may present problems for the statistical analysis, due to possible variations of the lithology within such a window. In our study, the average window length for the analysis was determinated by the petrophysicist as 150 ft, which corresponds to 32 s in the seismic traces. As shown in [13], we expect the ability of the method that separates different rock types to improve as the central frequency of the signal increases. These results suggest that the use of seismic data after deconvolution should produce clearer separations between different rock types. Nevertheless, deconvolution was not applied to the data we used.

We assume that the frequency content of the seismic data does not change significantly within the interval of interest, since drastic changes in frequency content may produce large changes in the fractal behavior that are not related to changes in rock type. For this reason, the method proposed in this paper will work best when the depth may introduce changes in frequency content due to signal attenuation.

In our study, the target interval was flat at a depth of 9000 ft. The average frequency with this interval was 30 Hz.

Fig. 1 shows typical γ-ray logs and Fig. 2 seismic traces corresponding to wells of each of the considered lithologies. The γ-ray logs show considerable differences when changing the predominant rock type, but seismic data recorded in the same positions do not show the same obvious differences. However, as we show in Sections IV–VI, seismic data do respond differently depending on the predominant rock type where they travel producing traces with the different energy levels. Our method is able to detect those subtle differences.

IV. Characterization of the Lithology

Our method is based on posing the characterization of the lithology of the reservoir as a problem of statistical model comparison. After calculating the continuous-wavelet decomposition of a particular signal, we obtain a collection of coefficients α_j. To illustrate the process of estimating α, we considered a particular seismic trace and took two values of b: 40 and 450 ms, regressing in each case $\log_2([\mathrm{CWT}_u(a,b)]^2)$ on $\log_2 a$. For each straight line, we obtained one slope -3.1615 and -3.0585, respectively, i.e., one estimate of the α coefficient. The results are shown in Fig. 3. Since there are as many b values as data points in the signal, we obtain a transformation of the signal in terms of energy power coefficients α. The previous process was repeated for each of the seismic traces and well logs.

Fig. 4(a) shows the estimated densities of the coefficients corresponding to the γ-ray logs of sand and gravel wells. An analogous plot is shown in Fig. 4(b) for seismic traces around those wells. We observe, in all cases, a rather picked shape, that suggests the use of a double-exponential density. Also, it is apparent

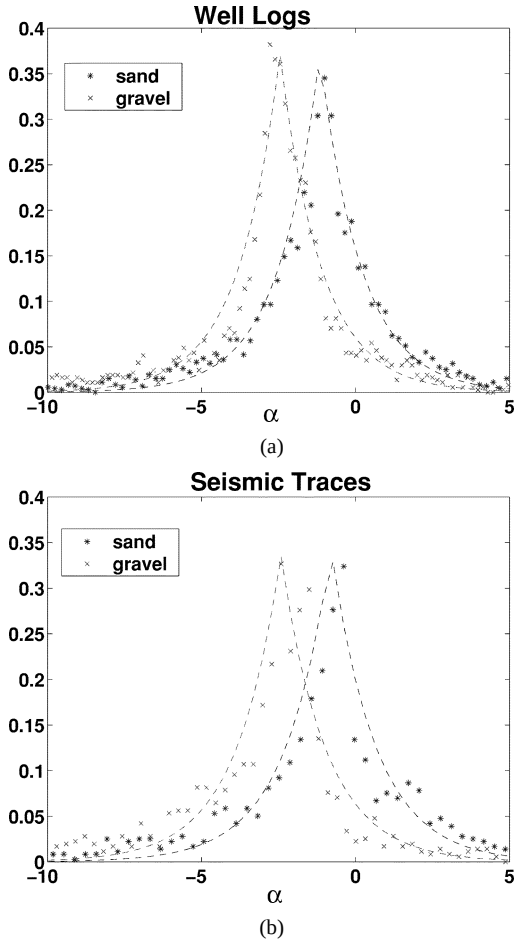

Fig. 4. Frequency distribution for the power coefficients α for (a) γ-ray logs and (b) seismic traces. Dotted lines represent double-exponential densities fitted by maximum likelihood.

from Fig. 4 that the locations of the densities change with the lithology.

To assess the dependence of the previous results on the type of wavelet function, we considered a Haar basis, a Mexican Hat, and a Gaussian. In all cases, we obtained coefficients with a picked density that appeared to be different for points that correspond to different lithologies. The results presented in this paper are based on the use of the Haar's basis.

Based on the shape of the distribution shown in Fig. 4, we propose the following model:

$$f\left(\alpha_{\mathbf{j}}|\mu_i,\sigma_i\right)=\frac{1}{\sqrt{2}\sigma_i}\exp\left\{-\frac{\sqrt{2}\left|\alpha_j-\mu_i\right|}{\sigma_i}\right\},\qquad i=1,2 \tag{2}$$

where $i=1$ corresponds to sand and $i=2$ to gravel.

Grouping the power coefficients that correspond to all wells with the same lithology, we can obtain maximum-likelihood estimators of the locations μ_i and scales σ_i of the proposed double-exponential distribution. The results are shown in Table I.

We observe that the location parameters seem to differ, whilst the scale parameters appear to be equal. In order to formally assess the significance of those differences we consider the following four possible models for the distribution of the coefficients:

$$\begin{array}{llll} M_1: & \mu_1=\mu_2 & M_2: & \mu_1\neq\mu_2 \\ & \sigma_1=\sigma_2 & & \sigma_1=\sigma_2 \\ M_3 & \mu_1=\mu_2 & M_4: & \mu_1\neq\mu_2 \\ & \sigma_1\neq\sigma_2 & & \sigma_1\neq\sigma_2. \end{array}$$

TABLE I
ESTIMATED POWER PARAMETERS FOR EACH LITHOLOGY

	γ ray logs	seismic traces
$\hat{\mu}_1$	-0.6500	-0.800
$\hat{\mu}_2$	-2.4550	-2.4002
$\hat{\sigma}_1$	1.5543	1.7013
$\hat{\sigma}_2$	1.5953	1.7300

It is apparent from Fig. 4 that the location of the densities changes with the lithology, so we expect that the selected model will be M_2, which corresponds to densities with different location parameters. Note that in Table I the absolute value μ_1 (the mean of the double-exponential probability model) is larger for sand than for gravel, which indicates that the fractal dimension of signals from different rock types is different. However, the physical reasons that explain these difference are still unknown and need more research.

We also applied our methodology to a dataset consisting of two resistivity logs as well as two density logs, from two different wells. These were recorded with the aim of characterizing a clastic, Eocene reservoir in Lake Maracaibo, Venezuela, located at a depth of 12 300 ft. These wells are located in areas that penetrate sandy and clay-contaminated sand environments, respectively. We obtained analogous results: the distribution of the scaling exponents corresponded to double-exponential distributions with significant differences in the mean parameter, and the method was able to discriminate the lithologies.

V. MODEL SELECTION

In order to formalize the comparison between the two distributions proposed in the previous section, we consider the more general case where there are k possible models $M_1,\ldots,M_k$ for a set of data $\mathbf{x}$. The ith model M_i corresponds to a density $f_i(\mathbf{x}|\theta_i)$, $\theta_i\in\Theta_i$, and $\pi_i(\theta_i)$ is a distribution that expresses prior knowledge on θ_i, while $p_i, i=1,\ldots,k$, $\sum_{i=1}^{k}p_i=1$ corresponds to the prior probability that we assign to model M_i before observing any data.

In our case, $\mathbf{x}$ corresponds to vector α in (2) and $\theta=(\mu,\sigma)$. Following a Bayesian approach, probabilities p_i can be updated using the *Bayes factor*, which is defined as

$$B_{ij}(\mathbf{x})=\frac{m_i(\mathbf{x})}{m_j(\mathbf{x})}$$

where

$$m_i(\mathbf{x})=\int_{\Theta_i}f_i\left(\mathbf{x}|\theta_i\right)\pi_i\left(\theta_i\right)d\theta_i$$

is the *marginal distribution* of $\mathbf{x}$ under model M_i. Then the posterior probability of model M_i after observing the data is

$$\Pr\left(M_i|\mathbf{x}\right)=\left[1+\sum_{j\neq i}\frac{p_j}{p_i}B_{ji}\right]^{-1}.$$

TABLE II
POSTERIOR PROBABILITIES OF MODELS

$P(M_i\|\alpha)$	γ **ray logs**	**Seismic traces**
M_1	7.3614e-7	3.7961e-11
M_2	0.9618	0.9717
M_3	2.4336e-8	9.8914e-13
M_4	0.0382	0.0283

The Bayes factor B_{ij} can be interpreted as the posterior odds ratio of M_i to M_j. It describes the weight of evidence in the data in favor of the model M_i, (see [21]). Assuming a 0–1 loss function, the optimal Bayesian selection corresponds to the model with greater posterior probability.

Difficulties arise if the prior $\pi(\theta_i)$ is specified to represent very weak information about θ_i. If the parameter space is unbounded, we cannot specify $\pi(\theta_i)$ as a uniform distribution, and thus the prior distributions are generally improper and depend on an arbitrary constant $\pi_i^N(\theta_i) = c_i p_i^N(\theta_i)$. The resulting Bayes factor

$$B_{ji}^N = \frac{c_j}{c_i} \frac{\int_{\Theta_j} f_j(\mathbf{x}|\theta_j)\, p_j^N(\theta_j)\, d\theta_j}{\int_{\Theta_i} f_i(\mathbf{x}|\theta_i)\, p_j^N(\theta_i)\, d\theta_i}$$

is undefined (the constants are not canceled) and cannot be used for model selection.

A possible solution to this problem is to approximate the Bayes factor with the following criteria:

- *Akaike's Information Criterion*

$$\mathrm{AIC} = -2\log\left(\frac{m_i}{m_j}\right) = 2\log(\lambda(x)) + (q_i - q_j) \quad (3)$$

- *Bayesian Information Criterion*

$$\mathrm{BIC} = -2\log\left(\frac{m_i}{m_j}\right) = 2\log(\lambda(x)) + (q_i - q_j)\log(n) \quad (4)$$

where $\lambda(x) = (L(\Theta_i|\mathbf{x}))/(L(\Theta_j|\mathbf{x}))$ is the classical likelihood ratio test statistic (e.g., see [22]), and q_i the number of parameters in model M_i. Both criteria can be used to compare models M_i and M_j.

The AIC was proposed to counteract the tendency of classical tests to favor the more complex model if the amount of data is large. On the other hand, the BIC criterion achieves an even more radical adjustment $(p_i - p_j)\,\log(n)$, which increases with n. For details see [23] and [24]. Other approaches have been proposed in [25].

We compare the four models proposed in Section IV using BIC and obtain the results shown in Table II. Model M_2 has greater posterior probability in both cases: well logs and seismic traces, suggesting that a characterization of the two lithologies can be done by the location parameters of the double-exponential distribution.

VI. CLASSIFICATION

The results obtained in the Section V show that a successful characterization of the lithology can be done in terms of the distribution of the power parameter. We can now use this fact to classify new observations: i.e., suppose observations at a new location become available; then, after performing the transformation of the data, we have a new sample of power parameters. The classification procedure consists of calculating the probability that this sample belongs to one of the two double-exponential distributions.

The general problem of classifying a new observation $\mathbf{y}$ within a collection of k possibles classes $C_i, i = 1\ldots, k$, after observing a sample $\mathbf{x}$, can be stated as follows. Suppose $\mathbf{y}$ has density $f_i(\mathbf{y}|\theta_i)$ if it belongs to class C_i, and let p_i and $\pi_i(\theta_i)$ be, as before, the prior probability of class C_i and the prior distribution of θ_i, respectively. Then the posterior density of θ_i

$$\pi_i(\theta_i|\mathbf{x}) \propto f_i(\mathbf{x}|\theta_i)\ \pi_i(\theta_i)$$

can be used to compute the predictive density of a future observation $\mathbf{y}$ given the data $\mathbf{x}$, under class C_i

$$\pi_i(\mathbf{y}|\mathbf{x}, C_i) = \int f_i(\mathbf{y}|\theta_i)\, \pi_i(\theta_i|\mathbf{x})\ d\theta_i. \quad (5)$$

Thus, the posterior probability that $\mathbf{y}$ belongs to the class C_i is

$$\Pr(C_i|\mathbf{y}, \mathbf{x}) = \frac{p_i \pi_i(\mathbf{y}|\mathbf{x}, C_i)}{\sum_{l=1}^{k} p_l \pi_l(\mathbf{y}|\mathbf{x}, C_l)} \quad (6)$$

and we can assign $\mathbf{y}$ to the class C_i for which $\Pr(C_i|\mathbf{y}, \mathbf{x})$ is maximum. In the double-exponential case, using the prior $\pi_i(\mu_i \cdot \sigma_i) \propto 1/\sigma_i$, which is known as the Jeffrey rule and is accepted as a reference noninformative prior (see [21]), we obtain that the predictive density is

$$\pi_i(\mathbf{y}|\mathbf{x}, C_i) = \frac{\Gamma(n-1)}{2^n} \sum_{\substack{j=0 \\ j \neq (n/2)}}^{n} \frac{n+1}{2j-n} \cdot \left[\frac{1}{S_j^{n-1}} - \frac{1}{S_j^{n-1}}\right] \frac{z_{((n/2)+1)} - z_{(n/2)}}{S_{(n/2)}^n} \quad (7)$$

where $\mathbf{z} = (\mathbf{x}', \mathbf{y}')'$, n is the dimension of $\mathbf{z}$ equal to $n_x + n_y$; n_x and n_y are the lengths of vectors $\mathbf{x}$ and $\mathbf{y}$ respectively, $z(i)$ denotes the order statistic of the sample $x_1, x_2, \ldots, x_n$ that is the sampled values placed in ascending order and $S_j = n\bar{z}_{(n)} - 2j\bar{z}_{(j)} + 2jz_{(j)} - nz_{(n)}$ with $\bar{z}_{(k)} = k^{-1}\sum_{i=1}^{k} z_{(k)}$, $\bar{z}_{(0)} = 0$, and $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\, dt$.

The procedure defined by (5)–(7) can be used to classify observations obtained from either well logs or seismic traces. To map the information in these two types of data, we note that they are based on different tools and procedures, and so it is reasonable to consider them conditionally independent. We modify the numerator in (6) by letting

$$\pi_i(\mathbf{y}|\mathbf{x}, C_i) = \pi_i(\mathbf{y}|\mathbf{x_w}, C_i)\, \pi_i(\mathbf{y}|\mathbf{x_t}, C_i). \quad (8)$$

Locations where seismic traces are available conform a grid. We calculated the posterior probability of sand or gravel in each point of the grid using the predictive density in (7) with the information of the seismic traces and the well logs. For each location on the grid, we considered the 25 nearest neighbors to obtain the mean trace and transformed it to obtain the corresponding α coefficients. In such a way, we obtained vector $\mathbf{y}$ in (5). Vector $\mathbf{x_t}$ in (8) is given by the mean traces around the locations of the wells, and $\mathbf{x_w}$ is the vector of wells logs. We considered initial probabilities equal to 0.5 for both types of lithologies. The map

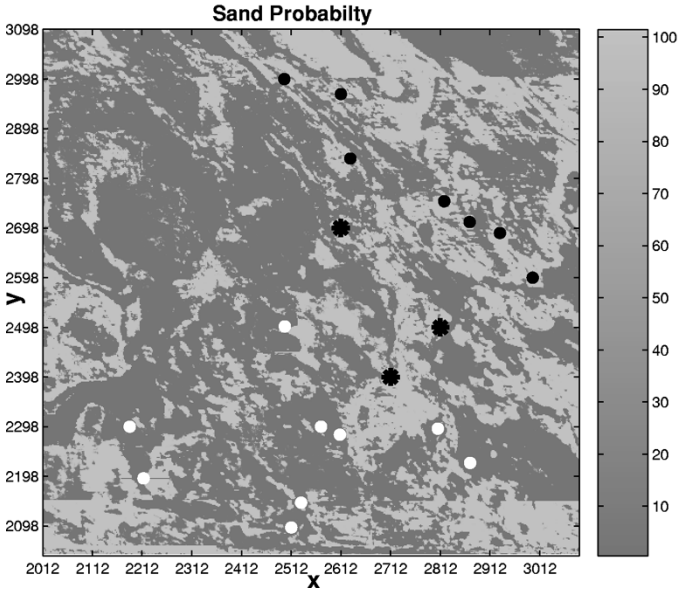

Fig. 5. Facies classification map using the posterior probability of sand based on seismic traces and well logs at points that are marked.

TABLE III
ESTIMATED PROBABILITY OF MISCLASSIFICATION

Lithology	γ ray logs	seismic traces
Gravel wells	0	0.11
Sand wells	0	0.08

obtained with this procedure is shown in Fig. 5. The points that appear in the map correspond to the locations of wells for which the lithology is known and in which the classification was correct.

In order to validate the classification procedure, we considered each point of the reservoir where γ-ray logs are available. For these the lithology is fairly well studied. We calculated the probability of sand and gravel using (6), where $\mathbf{y}$ was taken as the γ-ray log of a given well and $\mathbf{x}$ as the logs of all remaining wells, in a cross-validating fashion. The proportion of misclassified wells calculated is given in the left column of Table III. A similar analysis was performed using the mean seismic trace at each well, producing results reported in the right column of Table III.

VII. CONCLUSIONS

We have presented a methodology to characterize the lithology of a zone of interest in a reservoir. The method is based on an estimation of the power energy coefficient of signals corresponding to γ-ray logs as well as seismic traces penetrating the rock. In both cases, we find significant differences in the mean value of the power coefficient.

The characterization is extended in a Bayesian fashion to obtain a classification procedure that uses the information contained in both types of signals simultaneously. The results are checked, for well locations, using cross validation. For other points in the reservoir, we present a map based on the probabilities of each type of lithology, which is highly consistent with geophysical knowledge of the area.

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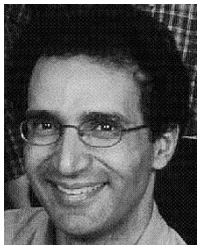

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