Tomographic String Inversion

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SUMMARY

When square pixels are used to parameterize the slowness field in traveltime tomography, the problems of discretization for inversion purposes and discretization for display purposes are inextricably mixed. A "high resolution" result demands many pixels or model parameters, thus burdening tomography with the inversion of a large and sparse projection matrix though simplifying the display problem by using regions of constant slowness. In this paper, we present a method of separating the tomography problem into two distinct steps - first inversion, then imaging. To reduce the complexity of the inversion step, we select a more natural set of pixels which are derived from the raypaths used to model the process of creating the traveltime data. The support of the string function is the raypath itself, thus the strings and raypaths are orthogonal (except when both equal the same line) and the projection matrix is diagonal and invertable in closed form. We are then left to image, i.e., synthesize the inversion result for display purposes. The method is tested on cross-well synthetic and field data requiring iterative curved raytracing. It is demonstrated to be a fast and robust technique of traveltime tomography.

INTRODUCTION

A solution to the well-known non-linear inversion problem of traveltime tomography in strongly refracting media involves three distinct steps: first, we must pick observed traveltimes; second, we calculate traveltimes for an assumed slowness model; third, we invert a matrix equation where the data is given by the traveltime residuals (calculated minus observed) to obtain corrections to the assumed slowness model. Steps two and three are performed iteratively and constitute linearization of the original non-linear problem of finding both the slowness field and the dependent raypaths. The iterations are usually stopped when an acceptable match between the calculated and the observed traveltimes is achieved.

The calculation of traveltimes, step 2, requires a mathematical model capturing albeit approximately the physical process of generating the data. For this purpose, we use ray tracing described by the equation

$$t_i = \int_\Omega S(x, z)\, \phi_i(x, z)\, dxdz. \qquad (1)$$

In general, the function $\phi_i(x, z)$ represents the 2D beampath which describes the area of the slowness field influencing the traveltime (Michelena and Harris,1990). However, in the formulation presented here, we take the function $\phi_i(x, z)$ to be nonzero only along the geometrical acoustic raypath, i.e., a line, thus equation (1) describes conventional raytracing in two spatial dimensions. The third step is the inversion of the residual traveltimes for perturbations to the slowness field. For the nonlinear problem, we separate the slowness into a known background $So(x, z)$ and a perturbation $\delta S(x, z)$. To set up the system of equations, we then parameterize the perturbations, that is, we devise a representation of the slowness field as a discrete superposition of known functions whose coefficients are to be determined by the inversion:

$$\delta S(x, z) = \sum_{j=1}^{N} \alpha_j\, \psi_j(x, z) \qquad (2)$$

With equation (2) representing the perturbations, the tomography problem is reduced to inverting the linear system of equations

$$\delta t = W\alpha, \qquad (3)$$

where W is the (MxN) projection matrix, α is the (Nx1) column vector of unknown coefficients parameterizing the slowness, and δt is the (Mx1) column vector of residual traveltime data. The elements of the projection matrix are given by the inner product of the basis function $\psi_j(x, z)$ with the raypath $\phi_i(x, z)$ both computed for the background medium:

$$W_{ij} = \int_\Omega \psi_j(x, z)\, \phi_i(x, z)\, dxdz \qquad (4)$$

When orthogonal pixels of constant slowness (Dines and Lytle, 1979; McMechan, 1988), are used for the basis functions $\psi_j(x, z)$, the projection matrix is typically very large and sparse, each element of it representing the segment of the ith ray projected on the jth pixel. These segment lengths are expensive to compute. Moreover, for the highly nonlinear problems encountered in seismic tomography the elements must be recomputed for each raytrace. Due to the large matrices, the solution of (3) by inversion of W is impractical, and (3) is often solved by iterative row-action methods (Censor, 1983). The choice of orthogonal pixels inextricably ties the inversion problem to the problem of sampling (and displaying) the estimated slowness field. A high resolution display requires smaller pixels, i.e., more model parameters, thus greatly increasing the dimensions of W and further increasing the cost of solving equation (3).

However, we are free to choose the basis functions and some choice other than orthogonal pixels of constant slowness may provide some advantages. This idea isn't new for splines and other parameterizations are often used to describe smoothly varying media. Michelena and Harris (1990) suggested using the beampaths as basis functions, thus forming a set of natural pixels wherein the matrix elements become the area of intersection of the ith beam with the jth beam. Conventional rays are used throughout our formulation; therefore, a judicious choice of basis functions might be to take $\psi_j(x, z) = \phi_i(x, z)$ or to use "strings" derived from the raypaths as the basis set.

STRING BASIS FUNCTIONS

When rays are used to model the traveltimes in equation (1) and strings are used for the basis set in equation (2), all off-diagonal elements of the projection matrix are identically zero and the diagonal elements simply reduce to the lengths of the rays. In this case, equation (3) simplifies greatly to become

$$\begin{pmatrix} l_1 & 0 \cdots & 0 \\ 0 & l_2 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots\cdots & & l_N \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \delta t_1 \\ \delta t_2 \\ \vdots \\ \delta t_N \end{pmatrix} \qquad (5)$$

Each of the coefficients α_i represents the average slowness along the path, i.e., $\alpha_i = \delta t_i / l_i$.. The fact that the off-diagonal elements of the projection matrix are zero implies that the correlation between different traveltime perturbations δt_i generated by equation (5) is zero. This is expected because once the raypaths are found in the medium $S_o(x, z)$, the traveltime perturbations are not affected by the slowness field in the neighborhood of the ray. If only one measurement is available, the result of the inversion is t / l (a constant slowness along the one string). For many different measurements the slowness estimate is just the superposition of the strings according to equation (2). A strength of the string method is that once a ray is traced, an update to the slowness model along that string can be calculated immediately. The cost of the inversion is only a small increment to the cost of raytracing.

Inversion along strings avoids two important and time consuming steps in tomography: 1) the computation of the matrix coefficients W_{ij} and 2) the inversion of the system of equations. However, a new problem arises: the imaging and displaying of the sum given by equation (2). Normally, with orthogonal pixels, we are defining the model description for display purposes as well as the correlation of points in the image with the data. Inversion along strings does not consider these aspects, a fact which keeps the inversion simple but shifts some burden toe the display. Decisions about filtering and display of the reconstruction are made after inversion, thus giving more freedom to perform post-inversion processing of $\delta S(x, z)$ for interpretation purposes.

The essential feature of the string inversion is that the basis function has nonzero support only along a line. Thus, the intersection of a basis function with a raypath is null except when they both identically equal the same line, thus resulting in a projection matrix which is diagonal. Moreover, the string need not have a uniform amplitude as formulated above. An interesting example is to define the string to have the geometry of the raypath and the amplitude distribution of the background slowness model $S_o(x, z)$ used to find the raypath. In this case, we have

$$\psi_j(x, z) = \phi_j(x, z) \bullet S_o(x, z) \qquad (6)$$

The coefficients parameterizing the slowness then become $\alpha_i = \delta t_i / t_i$ and are dimensionless. Furthermore, the inversion simplifies even further because the calculation of ray length is no longer required. Also, this dimensionless formulation makes it easier to enforce bounds on the magnitude of the perturbations $\delta S(x, z)$ by simply bounding α_i.

SYNTHETIC DATA

As described above, iterative tomography, i.e., inversion and imaging, involves two well-defined solution steps - raytracing and matrix inversion. Our implementation of these steps has some special features: first, we use vectorized ray tracing to calculate traveltimes, raylengths, and ray trajectories for a fan of rays. This avoids the time-consuming operation of linking the ray. For comparison purposes, the fan of observed traveltimes are interpolated onto the calculated raypaths (McMechan, Harris, and Anderson, 1988). Presently, we use 1D interpolation for each common-receiver-gather. We plan to improve upon this by using 2D interpolation of the pick data organized in the shot-receiver domain. String inversion is then performed on a super fine grid. After all rays are traced the result given by equation (2) is imaged, that is to say, equation (2) is filtered for display and for use as the background model for the next raytracing.

The string method was coded and tested on several synthetic examples simulating cross-well surveys. Results from one of these, a fault model with 12% velocity contrast, are shown in Figure 1. A synthetic dataset corresponding to a fault model was created using 100 source points spaced 10 feet apart. Each source radiated 50 rays in a fan +/- 45 degrees wide from horizontal. Five raytrace iterations were run from a constant velocity starting model. The results are shown in Figure 1. The mean absolute traveltime error after five iterations was 0.003 ms. The displayed images are sampled at 2.5-foot vertical and horizontal intervals. However, each velocity value is computed for a 10-foot by 10-foot cell about the sample point. Some artifacts are visible. However, these represent errors of less than 2% and are due mostly to the limited view and the asymmetry introduced by the raytracing.

FIELD DATA

The string tomography method was then tested on a real field dataset with over 5000 traveltimes acquired jointly by Amoco and Stanford University. The results for different inversions using two different starting models are shown in Figures 2 and 3. Again, the display is sampled at 2.5 feet with 10-foot square overlapping pixels. The mean absolute error for the two results after five iterations are 0.606 ms for the constant velocity starting model and 0.599 ms for the fault starting model. The two tomograms after five iterations are very similar and more importantly lead to a common interpretation. These data were also inverted using a square pixel method that produced a similar tomogram (Harris and Tan, et. al, 1990), where an interpretation can be found also.

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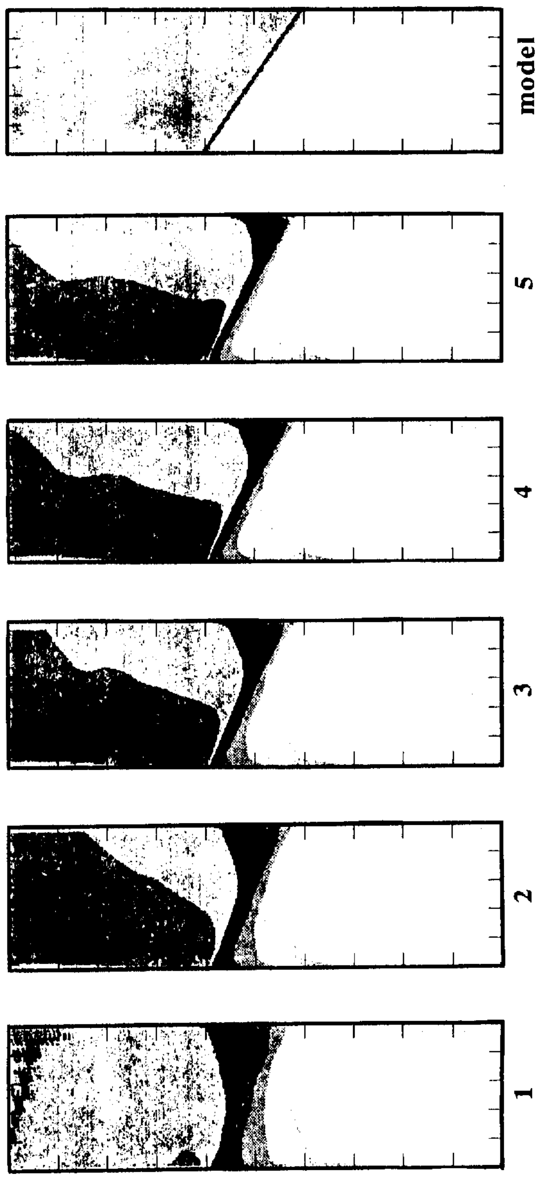

Fig. 1. Five iterations of the string inversion. Distance between wells is 250 feet. A depth interval of 1000 feet is displayed.

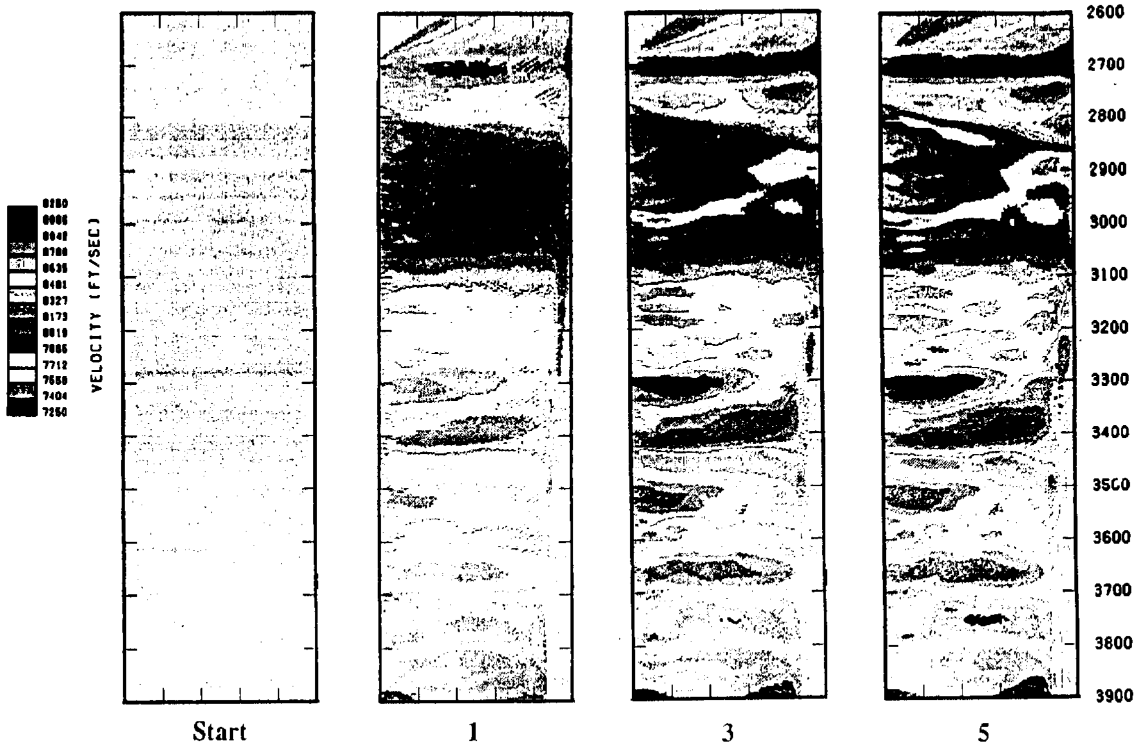

Fig. 2. Three of five iterations of the string inversion applied to field data. Distance between wells is approx. 250 feet. A depth interval of 1300 feet is displayed. The unimaged zone to the lower right of figures is outside deviated well.

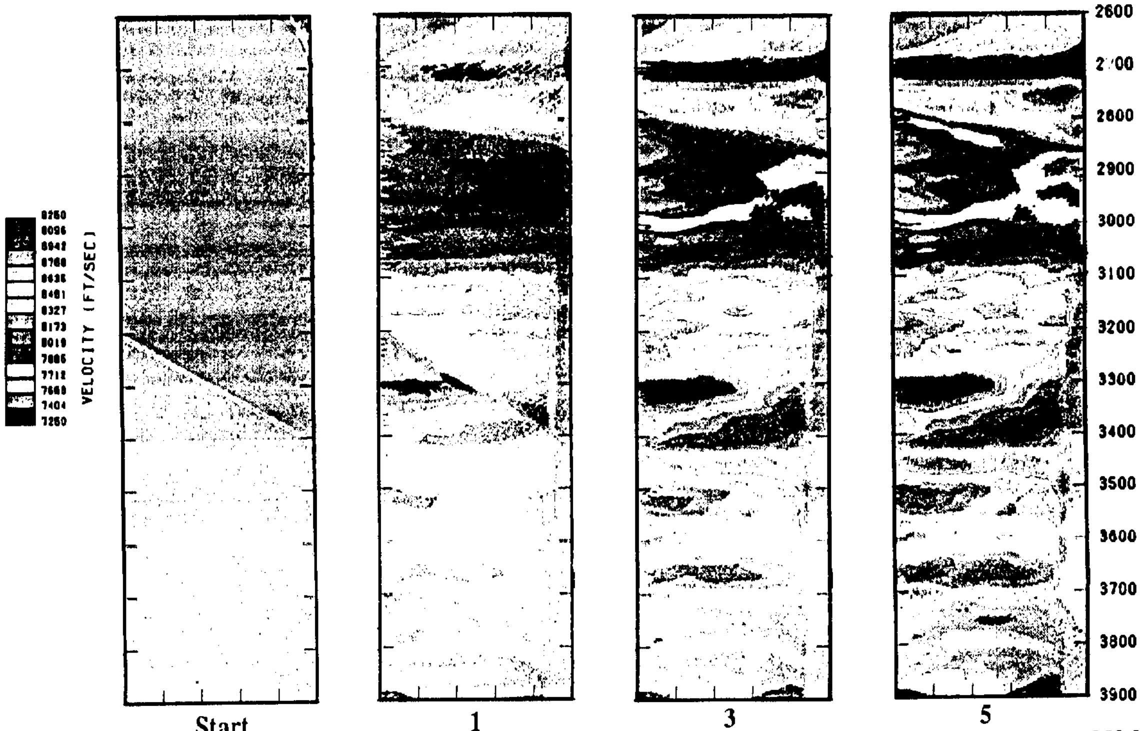

Fig. 3. Three of five iterations of the string inversion applied to field data. Distance between wells is approx. 250 feet A depth interval of 1300 feet is displayed. The unimaged zone to the lower right of figures is outside deviated well.